Eyes on Maths

A Visual Approach to Teaching Maths Concepts

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EDUCATION

Contents

CHAPTER 1

Background

THIS BOOK IS FOUNDED on two bodies of research, one focusing on the vital role visualisation plays in teaching and learning mathematics and one focusing on the importance of ensuring that students deeply understand the mathematical concepts with which they must deal.

VISUALISATION IN MATHS

When children are young, we use picture books not only to introduce them to the power of books and to the value and joy of being literate but also because visual images are powerful in helping us make sense of our world. Adams and Victor (1993) suggest that vision is the most important source of information about the world. Our embrace of the newest technologies, with the prominence of tablet computers and YouTube, indicates that we have become a highly visual culture in which pictorial images have begun to supplant the printed word as our preferred way of navigating the world.

Sadoski and Paivio (2001) have shown the critical role of visualisation in the domain of reading, and it seems reasonable that the same would be true in the development of mathematical thinking.

Another researcher who has addressed the power of visual imagery is Edward R. Tufte (2001). Although he speaks primarily on the power of visual presentation of statistical data, his ideas can be easily generalised to other aspects of mathematical information.

Consider the power, for example, of showing why $3 \times 4 = 4 \times 3$ using the visual image below as compared with the more symbolic algebraic definition that appears on the next page.

A Visual Argument for Why $3 \times 4 = 4 \times 3$

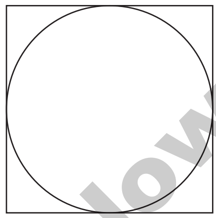
I see 3 rows of 4 bananas, but I also see 4 columns of 3 bananas.

Since $m \times n$ represents m groups of n items, I am seeing both 3×4 and 4×3 when I look at the same thing.

An Algebraic Argument for Why 3 × 4 = 4 × 3

$$\begin{aligned} 3 \times 4 &= 4 + 4 + 4 \\ &= 3 + 1 + 3 + 1 + 3 + 1 \\ &= 3 + 3 + 3 + (1 + 1 + 1) \\ &= 3 + 3 + 3 + 3 \\ &= 4 \times 3 \end{aligned}$$

Similarly, consider the power of showing why the circumference of a circle is less than four times its diameter using the picture below, where the circumference of the circle is clearly less than the perimeter of the square, which has each side length equal to the circle's diameter.



In fact, an interesting book at the secondary level is called *Proofs Without Words* (Nelsen, 2000). In its focus on higher-level mathematical thinking, it shows how ideas are often more meaningfully demonstrated by using pictures as opposed to using symbols.

Rowan and Bourne (1994) have pointed out that understanding mathematical concepts is supported by children's ability to see how those concepts play out. This often involves the use of manipulatives, and it may be that it is not just the kinesthetic, but also the visual, aspect of using manipulatives that is what is relevant. Students can maintain visual images as they work through more abstract descriptions of ideas.

Murphy (2007) points out that it is "well thought-out and carefully developed visual/verbal co-expression of content" (p. 3) that is essential for visual learning.

FOCUSING ON THE IMPORTANT MATHS

There has been an increasing emphasis on identifying "focal points" that should be addressed with students when teaching mathematics. This is evidenced in the *Curriculum Focal Points* series developed by the National Council of Teachers of Mathematics (2006), as well as the NCTM series *Developing Essential Understanding* . . . (e.g. Dougherty, Flores, Louis, Sophian & Zbiek, 2010). The latter series helps teachers see the most important ideas in each of the topics addressed. Most recently, the effort to develop the Australian Curriculum: Mathematics has been an attempt to create a mathematics curricula that is more focused and more coherent than ever before. The notion is that too many people view mathematics as a checklist of individual skills and concepts students must meet and master instead of seeing it as a connected web with a relatively small number of highly significant ideas that are visible in many individual situations.

At this point, the objective in developing methods for teaching mathematics is to focus not just on isolated topics but also on what it is about any topic that is important to emphasise. For example, Dougherty et al. (2010) list both big ideas and essential understandings for number and numeration for Pre-K to Grade 2. Rather than just suggesting how high students should count, or the size of the numbers they should add, these authors list concepts students should master in relation to number and numeration, including

- Recognising that quantities can be compared without counting
- Recognising that the size of a unit determines the number of times it must be iterated to count
- Recognising that inherent in a place value system is the use of units of different size

Ideas such as these are addressed in the visuals and questions posed about them in the book.

BUILDING MATHEMATICAL COMMUNICATION

Building a maths talk learning community is an integral step in improving mathematical understanding in a classroom. For the teacher, this involves a progression from simply asking factual short-answer questions to focusing on mathematical thinking and involving students as active participants in the development of concepts in the classroom.

Bruce (2007) points out that to cultivate valuable classroom communication, one of the important elements is the use of rich tasks. Hufferd-Ackles, Fuson and Sherin (2004) suggest that a higher level of discourse is visible in a mathematics classroom when the teacher fosters an environment where students build on one another's explanations of ideas. Sullivan and Clarke (1992) discuss the importance of open-ended questions; they suggest the need for questions that require more than simple recall, that allow pupils to learn from the act of responding to the question, and that are open, with multiple possible correct answers. The questions associated with the visuals in this resource are such open and rich questions.

In the three chapters that follow, the value of visuals, the potential for high levels of communication, and a focus on important mathematical concepts will all be evident.