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Number and Operations

DIFFERENTIATED LEARNING activities in number and operations are derived from applying the NCTM process standards of problem solving, reasoning and proof, communicating, connecting, and representing to content goals of the NCTM Number and Operations Standard, including

- understanding numbers, ways of representing numbers, relationships among numbers, and number systems
- understanding meanings of operations and how they relate to one another
- computing fluently and making reasonable estimates (NCTM, 2000)

TOPICS

Before differentiating instruction in number and operations, it is useful for a teacher to have a sense of what number concepts students typically bring to a grade level and how concepts in number and operations tend to develop after that level. The NCTM Curriculum Focal Points (NCTM, 2006) are one source of this knowledge, as are state or local standards and research findings.

Students move from working comfortably with relatively small numbers concretely to working with whole numbers up to 1,000 symbolically to working with fractions, decimals, and larger whole numbers (Small, 2005a). They move from solving problems involving addition, subtraction, multiplication, and division by counting to solving problems using strategies and learned and invented procedures; as students develop mathematically, they flexibly use more efficient procedures and strategies.

Prekindergarten–Grade 2

Within this grade band, students move from counting and comparing very simple numbers, usually 10 or less, to counting, comparing, modeling, and interpreting numbers up to 1,000 using place value concepts. They move from counting to determine sums in simple joining situations and differences in simple subtraction situations to thinking more formally about adding and subtracting. Increasingly, as they move through the grade band, students use a variety of principles, strategies, and procedures with increasing efficiency to add and subtract and to solve problems requiring addition and subtraction.

Grades 3–5

Within this grade band, students begin to focus increasingly on multiplying and dividing whole numbers using a variety of strategies to calculate and estimate **products** and then **quotients**. They commit multiplication and related division facts to memory, become more fluent with **algorithms** for multiplying and dividing multidigit whole numbers, and solve problems that represent a variety of meanings of multiplication and division.

Students also begin to develop a greater understanding of fractions, first modeling, representing, and comparing them, and later using **equivalent fractions** to simplify the task and to add and subtract fractions. They start to use decimal notation to represent numbers that have fractional parts and learn to add and subtract decimals to solve problems that require these operations.

Grades 6–8

Within this grade band, students extend their understanding of fractions and decimals to situations involving multiplication and division of these values. They increasingly work with ratio and **proportion**, particularly, but not exclusively, in percentage situations. They also begin to work with more abstract values, including **negative integers**, **exponents**, and **scientific notation**.

THE BIG IDEAS FOR NUMBER AND OPERATIONS

Coherent curricula in number and operations that meet NCTM content and process standards (NCTM, 2000) and support differentiated instruction can be structured around the following big ideas:

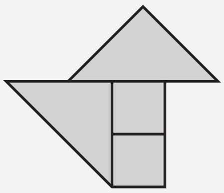
- There are many ways to represent numbers.
- Numbers tell how many or how much.
- Number benchmarks (referent numbers that are familiar and meaningful, such as 10, 25, 100, 1,000, etc.) are useful for relating numbers and estimating amounts.
- By classifying numbers (e.g., in terms of how many digits they have, whether they are odd or even, etc.), conclusions can be drawn about them.
- The patterns in the **place value system** can make it easier to interpret and operate with numbers.
- It is important to recognize when each operation (addition, subtraction, multiplication, or division) is appropriate to use.
- There are many different ways to add, subtract, multiply, or divide numbers.
- It is important to use and take advantage of the relationships between the operations in computational situations.

The tasks set out and the questions asked while teaching number and operations should be developed to reinforce these ideas. The following sections present numerous examples of application of open questions and parallel tasks in development of differentiated instruction in these big ideas across three grade bands.

BIG IDEA. New shapes can be created by either combining or dissecting existing shapes.

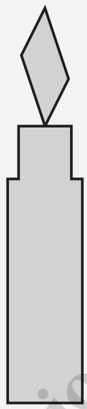
Use your tangrams.

Option 1:
Fill in the pieces:



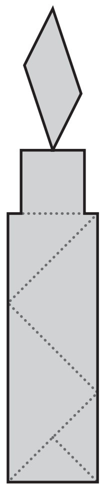
Option 2:
Use five tangram pieces to make a square.

Option 3:
Make this design:

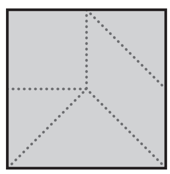


Option 1 is most suitable for students whose geometric concepts are less highly developed. If the diagram is made life size, students can simply experiment and try different pieces in different spots. In discussion at the conclusion of the work, those students are able to talk about how they fit the shapes in, what clues they used, and so on.

Option 3 is a mathematically more sophisticated activity than ***Option 1*** because no internal lines are presented. Its complexity, too, can be varied based on whether the diagram is life size. A life-size diagram will always be easier to work with because students will be able to fit pieces into the outline. The diagram at the right shows the tangram candle with outlines of the component pieces.



Option 2 is the most open. An even more open alternative would be to simply ask students to use any number of tangram pieces to make a square, offering some very simple solutions. By requiring exactly five pieces, the task poses more challenge to students than it would in its simplest form. The diagram at the right shows a square made up of five tangram pieces, with outlines.



The follow-up discussion could invite students, no matter what task they chose, to describe how they went about solving their problem.

Variations. ***Option 1*** can be adjusted to be slightly more difficult by using a life-size diagram with no internal lines shown.

BIG IDEA. The same object can be described by using different measurements.

How can you measure a pumpkin?

A student can think of many different ways to measure a pumpkin: width, height, weight, or circumference. Leaving the question open allows each student to choose a measurement on which to concentrate.

Class discussion about the various types of measurements students used will allow them to recognize the fact that there is usually more than one way to measure an object.

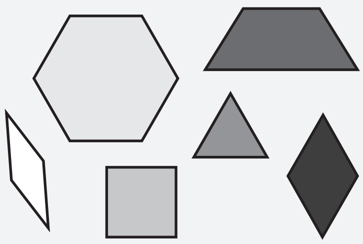
Variations. Rather than a pumpkin, another interesting object can be selected. For example, students can be asked to measure a bag of popcorn, a car, or a container.

TEACHING TIP. Items to be measured can be chosen based on themes being pursued in other subject areas of instruction, special occasions, or holidays.

BIG IDEA. The numerical value attached to a measurement is relative to the measurement unit.

Choose a number of **pattern blocks** of the same color.

- How can you put them together around a single point so that there is no empty space?
- What does that tell you about how big the angles are that you put together?



Many times the teaching of angles is introduced with standard units. There is value, however, in thinking about nonstandard units to get across the concept of angle measurement without the additional complication of trying to convey what a degree is. Putting together many copies of the same angle, as is suggested in this task, helps students think about an angle as a nonstandard unit.

Allowing students to choose which shape they use gives them the chance to pick a simple angle (such as a right angle in the square) or a more complex angle (e.g., one of the trapezoid angles). A connection to degrees can also be made. Students will notice that 3 hexagons can be put together, so the angle measure is 360 ÷ 3 = 120°; 6 triangles can be put together, for an angle measure of 360 ÷ 6 = 60°; 4 squares can be put together, so the angle measure is 360 ÷ 4 = 90°; either 3 or 6 **rhombuses** can be put together, depending on which angles are used; and 12 of the thin kites can be put together if the small angles are used, for an angle measure of 360 ÷ 12 = 30°.

✷ **BIG IDEA. Units of different sizes and tools of different types allow us to measure with different levels of precision.**

> Your grandfather is building you a miniature car out of wood. What units should he use to measure the parts of the car? Why?

Students might have differing opinions on what a miniature is—is it very tiny, such as one that would fit in a pencil case, or is it the size a baby could sit in? This decision on scale could affect the student's choice of unit.

Many students relate the size of a unit only to the size of what is being measured; they would automatically use a small unit for a small object and a large unit for a large object. Although doing this makes sense, the teacher should help students understand that the precision needed in the measurement is also important. Even when measuring something that is longer than a ruler, one might decide to measure in millimeters because of a need to be precise. Students should consider whether the construction project would or would not require this level of precision.

The flexibility in what the miniature could look like and how precise the measurements would need to be makes the question open to a broad range of students.

Variations. The task can be varied by asking students to consider what tool or unit might be used to measure a long distance, for example, the distance from one side of the school to the other side.

✷ **BIG IDEA. Knowledge of the size of benchmarks assists in measuring.**

> A container holds about 4 gallons. Describe its size in other ways.

Some students will answer the question by relating the 4 gallons to familiar, known containers. For example, if a student knows that a kitchen pot at home holds 1 gallon, he or she can imagine another pot that holds 4 times as much.

Other students will answer the question by suggesting width, depth, or height measurements that are reasonable for a container of this size.

Discussing the various approaches to the problem will help all students understand how knowing the size of one item can assist in determining the size of other items.

> Choose a pattern block. Create a shape made of 20 of your blocks.
>
> Now choose another block. How many blocks of this type will you need to make a shape that takes up the same space?

This question is designed to allow students to create their own measurement benchmarks. Students can choose to make their first shape with small blocks so that they will need fewer of the alternate blocks, making it easier to estimate. Or they might choose the challenge of using larger blocks at first.

One solution might be, for example, that the space occupied by 20 yellow blocks is the same as the space occupied by 40 red blocks. Another is that the space occupied by 20 triangles is the same as the space occupied by 10 rhombuses.

TEACHING TIP. Pattern blocks are a useful tool for teaching concepts in number (particularly about fractions and ratio), geometry, and measurement.

> Why might it be useful to measure the length and width of a room by counting how many steps you need to get from one wall to the next?

Students begin to believe, often as a result of instruction, that once they know how to measure in inches, feet, yards, meters, or centimeters, there is no longer any point in measuring with nonstandard units. This question offers them the opportunity to confront that belief without actually telling them to do so. For example, a student might recognize that if his or her parents were measuring for carpet and only needed to estimate the cost, it would be quicker and easier to walk the distance than to take the time to measure exactly.

OPEN QUESTIONS FOR GRADES 6–8

✷ **BIG IDEA. The same object can be described by using different measurements.**

> A shape has an area of 200 square inches. What could its length and width be?

By not specifying what the shape is, this question is much more open than it would otherwise be. Some students will decide to use a square. They may struggle a bit because 200 is not a **square number**, but they may recognize that the square's length and width would be about 14 inches. Other students will choose a simpler strategy, using a rectangle of length 20 and width 10. Still other students will choose a shape with more sides, particularly if grid paper is provided that would allow them to experiment. Students should be encouraged to determine more than one answer so that they use a variety of formulas or alternate methods to create shapes.

Variations. The question can be varied by changing the area required or by adding stipulations to the type of allowable shapes. For example, it could be specified that the shape must have four or more sides or that at least two of its sides must be congruent. Another way to vary the question is to give a volume measurement rather than an area measurement.

TEACHING TIP. By providing the area of a shape and not specifying additional information about the shape, students are free to access area formulas that are familiar to them. During the follow-up discussion, they may be exposed to formulas with which they are less comfortable.

> You create the **net** of a 3-D figure and calculate its area. Then you fold the shape into the 3-D figure and calculate its volume.
>
> For example, for this net, the area is 16 square units and the volume is 4 cubic units.
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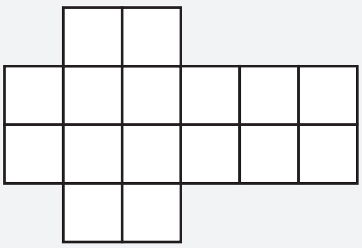
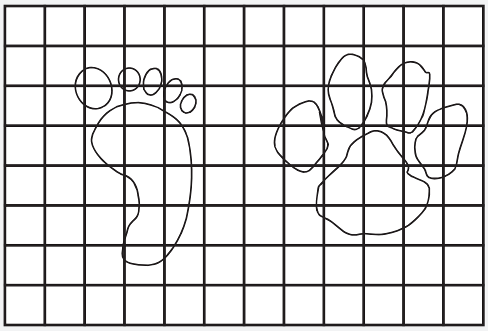
> 
>
> - Which is usually greater—the number for the area or the number for the volume? (This time the number for the area is greater.)
> - Why does that make sense?

Estimate the size of each footprint.



Option 1: Count the number of squares that the footprint fully covers. Count the number of squares that the footprint partially or fully covers. Select the number halfway between the two.

Option 2: Count the number of squares that are more than half covered by the footprint.

Option 3: Cover the footprint with aquarium gravel, one layer thick. Rearrange the gravel into a rectangle to estimate the area of the footprint.

All three options require students to consider what it means to estimate area. ***Option 3*** is more labor intensive but is probably the most comfortable for students because it relates the area of something irregular to something with which they are much more comfortable, a rectangle. ***Options 1*** and ***2*** merely require counting. ***Option 2*** demands that students estimate whether each square is more than half covered or not; many students will find this repeated decision making uncomfortable. ***Option 1*** is somewhat more straightforward, but why a number halfway between the two values is chosen may be somewhat of a mystery.

All three groups of students could be asked:

- *Is it easy to tell which of the footprints is larger just by looking at the grid?*
- *Did your estimates help you decide which was larger?*
- *What was your estimate for each?*
- *Why did those values seem reasonable to you?*

TEACHING TIP. It is important that students get a lot of experience in measuring irregular shapes, and not just regular shapes. Real life is full of both.

✷ **BIG IDEA. The use of standard measurement units simplifies communication about the size of objects.**

> ***Option 1:*** An object has an area of 12 square inches. What might it be?
>
> ***Option 2:*** An object has an area of 12 pattern block triangles. What might it be?

The difference between the two tasks relates to the student's comfort with standard area measurements. Some students might choose ***Option 2*** because it is easier for them to visualize the size of a pattern block triangle than a square inch.

Possible objects 12 square inches in size are a cordless phone laying flat on a table or the base of a large cup. Possible objects the size of 12 pattern block triangles include a flip cell phone or a roll of tape laying on its side.

For either task, the teacher could ask:

- *How much space would one unit use up? How do you know?*
- *How did you solve the problem?*

PARALLEL TASKS FOR GRADES 6–8

✷ **BIG IDEA. The same object can be described by using different measurements.**

> ***Option 1:*** One circle has a greater area than another. Does its circumference also have to be greater?
>
> ***Option 2:*** One rectangle has a greater area than another. Does its perimeter also have to be greater?

Some students who are new to the formulas involving circumference and area of a circle are likely to be attracted to ***Option 2***, using rectangles. What is interesting, however, is that the answer to ***Option 1*** is clearer than the answer to ***Option 2***. If one circle has a greater area than another, it has a greater radius and therefore a greater circumference. On the other hand, a rectangle with a greater area can sometimes have a greater perimeter and sometimes not. For example, a 6 by 10 rectangle has more area but less perimeter than a 1 by 30 rectangle, but a 4 by 5 rectangle has more area and more perimeter than a 1 by 2 rectangle.

Students completing either task could be asked:

- *How many combinations did you try?*
- *Can you be sure of the result with that many trials?*
- *How could a picture help someone understand your thinking?*
- *How might you have predicted what you found out?*

Each prism has whole number unit side lengths.

Option 1: A prism has a surface area of 126 square units and a volume of 90 cubic units. What could the dimensions be?

Option 2: A prism has a volume of 48 cubic units and a width of 4 units. What could the dimensions be?

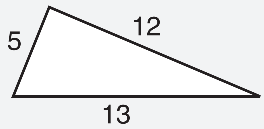
Option 2 will be easier for many students than ***Option 1*** because one of the three dimensions is provided. Students choosing ***Option 2*** will realize that the product of the length and height must be 12 square units and can simply list some possibilities. The challenge of ***Option 1*** requires students to simultaneously apply formulas for both surface area and volume.

Regardless of which option the students chose, it would be meaningful to ask:

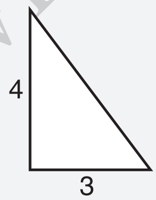
- *What is the relationship between the dimensions and the volume?*
- *What is the relationship between the dimensions and the surface area?*
- *Can you be sure that there are no other possible dimensions?*

Both triangles are right triangles.

Option 1: What is the area of the triangle?



Option 2: What is the perimeter of the triangle?



Option 1 is set up with all of the required information provided, as long as students recognize that orientation is irrelevant in describing the base and height of a triangle. Here the base could be viewed as the side with length 5 and the height as length 12; the area, therefore, is 30 square units. ***Option 2*** requires application of the Pythagorean theorem.

Whichever task was completed, students could be asked:

- *You calculated one measure of the shape. How did you do the calculation?*
- *Why does that method make sense?*
- *What if you had been asked to calculate a different measure?*
- *Would it have been equally easy?*