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Introduction

BACKGROUND

Most of us, when we think about math, think of what is generally considered *content.* We talk about working with fractions, or understanding place value, or solving equations. But in the background there have always been the fundamental mathematical practices and processes. Concepts such as reasoning, problem solving, recognizing structure, and modeling are not new to us.

One of the big changes instituted in the most recent curriculum revisions both in the United States and in Canada has been to bring these background processes to the foreground, not only to highlight their importance but also to help us see that if we focus explicitly on these processes, then content will be learned differently and may in fact be learned more effectively.

Much of this change began with the 1989 National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). In this document, notions of math as problem solving, as communication, and as reasoning and mathematical connections were explicitly discussed.

In 2000, NCTM updated the standards in the *Principles and Standards for School Mathematics* (NCTM, 2000) and specifically mentioned the following process standards:

- Problem solving
- Communication
- Representation
- Reasoning and proof
- Connections

These standards served as an underpinning for many state and provincial curricula.

More recently, the advent of the Common Core Standards in the United States has evolved these process standards into explicitly stated Standards for Mathematical

Practice. Canadian curricula still focus on process standards, although the names of the processes vary across the country.

The eight Standards for Mathematical Practice from the Common Core curriculum are listed here. They will be covered in detail in subsequent chapters.

COMMON CORE STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. (Related to the processes of problem solving and reflecting [one of the Canadian process standards listed below])
2. Reason abstractly and quantitatively. (Related to the processes of reasoning and representation)
3. Construct viable arguments and critique the reasoning of others. (Related to the processes of reasoning and communication)
4. Model with mathematics. (Related to the process of representation)
5. Use appropriate tools strategically. (Related to the process of representation)
6. Attend to precision. (Related to the process of communication)
7. Look for and make use of structure. (Related to the process of connections)
8. Look for and express regularity in repeated reasoning. (Related to the process of connections)

Many rich mathematical activities involve more than one of these standards at the same time, as will be noted in the later chapters.

CANADIAN CURRICULA PROCESS STANDARDS

In different provinces in Canada, these process standards are articulated in various curricula (Ministère de l'Éducation, Gouvernement du Québec, 2001; Ministry of Education Ontario, 2005; Western and Northern Canadian Protocol, 2006).

- Problem solving (or situational problem solving)
- Communication
- Representing
- Reflecting
- Technology
- Mental math and estimation
- Reasoning
- Connecting or Connections
- Selecting tools and strategies
- Visualization

The processes of problem solving, reasoning, representing, and connecting are directly linked to the Common Core Standards for Mathematical Practice.

The process of reflecting is an integral part of Mathematical Practice Standard 1, when students reflect on solutions, although reflecting could also include reflection on strategies, connections, or other aspects of mathematical work as well.

The processes of visualization and mental math and estimation are less directly addressed in the Common Core Standards for Mathematical Practice. Visualization is often embedded in problem solving (Practice Standard 1), constructing viable arguments (Practice Standard 3), and modeling (Practice Standard 4), but not necessarily. Mental math and estimation is often linked to looking for and making use of structure (Practice Standard 7), but not necessarily. Therefore, I have included an additional chapter in this resource (Chapter 9) to focus specifically on the processes of visualization and mental math and estimation.

MISCONCEPTIONS ABOUT THE STANDARDS FOR MATHEMATICAL PRACTICE AND THE MATHEMATICAL PROCESSES

Because of overall concerns with the Common Core, but particularly because the Standards for Mathematical Practice and the mathematical processes are "separated" from the other standards or outcomes/expectations in various curricula, many misconceptions can get in the way of using them as intended.

Some teachers believe that the intention is that all lessons must incorporate all standards for practice or all processes (Mateas, 2016). This is certainly not the case. Few, if any, lessons would include all of them, although many might include more than one. Although each standard and several processes are described separately in the ensuing chapters, I often reference how different standards can apply while working on the same problem. I also give a great deal of attention to when a particular standard or process might be appropriate.

Some believe it is possible to predetermine which standard or process will arise when students are confronted with a particular task. Although it is true that some tasks are more likely to elicit particular standards or processes than others, it is always up to the student what she or he brings to bear when working on a problem or in a mathematical situation.

Some teachers believe that the standards and processes are not taught; they just happen. Indeed, for some students, they will just happen. But there is definite benefit in articulating explicitly what these standards and processes are. This can only help students become more aware of their thinking and extend their thinking in new situations.

Some teachers believe that the standards or processes are not taught at the same time content is taught. This is not likely in that the descriptions of how to apply a standard or process require mathematical content in which to be embedded.

PROMOTING MATHEMATICAL THINKING

An overarching goal of teaching and making students aware of mathematical practices and processes has been to move students away from simply using algorithms for mathematical computation and to move them toward becoming mathematical thinkers. To this end, teachers have stepped back from simply telling students what to do and giving them practice at doing it. Instead they allow students to sometimes stumble as they move forward.

However, there is a fine line to be considered. I occasionally encounter teachers who feel that they should never give students any assistance. In fact, it is important for teachers to be attuned to when struggle is productive and when students need encouragement and prompts to their thinking in order to move forward. For this reason, I offer many descriptions in this resource of what questions to ask or what actions to take when students stumble.