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Introduction

Why and How to Differentiate Math Instruction

STUDENTS IN ANY CLASSROOM differ in many ways, only some of which the teacher can reasonably attend to in developing instructional plans. Some differences will be cognitive—for example, what previous concepts and skills students can call upon. Some will be more about learning style and preferences, including behaviors such as persistence or inquisitiveness or the lack thereof; whether the student learns better through auditory, visual, or kinesthetic approaches; and personal interests.

THE CHALLENGE IN MATH CLASSROOMS

Although many teachers of language arts recognize that different students need different reading material, depending on their reading level, it is often more challenging for teachers to vary the material they ask their students to work with in mathematics. The math teacher will more frequently teach all students based on a fairly narrow curriculum goal presented in a textbook, even though teachers have been asked to encourage a variety of strategies in the solution of problems. The teacher will recognize that some students need additional help and will provide as much support as possible to those students while the other students are working independently. **Differentiating instruction** in mathematics is still a relatively new idea. It is not easy in mathematics to simply provide an alternate book to read (as can be done in language arts). And a majority of teachers have never been trained to really understand how students differ mathematically. However, students in the same grade level clearly *do* differ mathematically in significant ways. Teachers want to be successful in their instruction of all students, and feel even more pressure to do so in the current social climate. Understanding differences and differentiating instruction are important processes for achievement of that goal.

The National Council of Teachers of Mathematics (NCTM), the professional organization whose mission it is to promote, articulate, and support the best possible teaching and learning in mathematics, recognizes the need for differentiation. The first principle of the NCTM *Principles and Standards for School Mathematics* reads, "Excellence in mathematics education requires equity—high expectations and strong support for all students" (NCTM, 2000, p. 12).

In particular, NCTM recognizes the need for accommodating differences among students, taking into account prior knowledge and intellectual strengths, to ensure that each student can learn important mathematics. "Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (NCTM, 2000, p. 12). This has been addressed more recently, after the widespread adoption of the Common Core Standards, in *Principles to Action* (NCTM, 2014), where a contrast is made between unproductive beliefs about access and equity in mathematics and more productive beliefs.

How Students Might Differ

One way that we see the differences in students is through their responses to the mathematical questions and problems that are put to them. For example, consider the task below, which might be asked of 3rd-grade students:

> In one cupboard, you have three shelves with five boxes on each shelf. There are three of those cupboards in the room. How many boxes are stored in all three cupboards?

Students might approach the task in very different ways. Here are some examples:

- Liam immediately raises his hand and simply waits for the teacher to help him.
- Angelita draws a picture of the cupboards, the shelves, and the boxes and counts each box.
- Tara uses addition and writes 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5.
- Dejohn uses addition and writes 5 + 5 + 5 = 15, then adds again, writing 15 + 15 + 15 = 45.
- Rebecca uses a combination of multiplication and addition and writes 3 × 5 = 15, then 15 + 15 + 15 = 45.

The Teacher's Response

What do all these different student approaches mean for the teacher? They demonstrate that quite different forms of feedback from the teacher are needed to support the individual students. For example, the teacher might wish to:

- Follow up with Tara and Dejohn by introducing the benefits of using a multiplication expression to record their thinking.

- Help Rebecca extend what she already knows about multiplication to more situations.
- Encourage Liam to be more independent, or set out a problem that is more suitable to his developmental level.
- Open Angelita up to the value of using more sophisticated strategies by setting out a problem in which counting becomes even more cumbersome.

These differences in student approaches and appropriate feedback underscore the need for a teacher to know where his or her students are developmentally to be able to meet each one's educational needs. The goal is to remove barriers to learning while still challenging each student to take risks and responsibility for learning (Karp & Howell, 2004).

WHAT IT MEANS TO MEET STUDENT NEEDS

One approach to meeting each student's needs is to provide tasks within each student's **zone of proximal development** and at the same time to ensure that each student in the class has the opportunity to make a meaningful contribution to the class community of learners. Zone of proximal development is a term used to describe the "distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86).

Instruction within the zone of proximal development allows students, whether through guidance from the teacher or through working with other students, to access new ideas that are close enough to what they already know to make the access feasible. Teachers are not using educational time optimally if they either are teaching beyond a student's zone of proximal development or are providing instruction on material the student already can handle independently. Although other students in the classroom may be progressing, the student operating outside his or her zone of proximal development is often not benefiting from the instruction.

For example, a teacher might be planning a lesson on multiplying a decimal by a whole number. Although the skill that is the goal of the lesson is to perform a computation such as 3×1.5, there are three underlying mathematical concepts that a teacher would want to ensure that students understand. Students working on this question should know:

- What multiplication means (whether repeated addition, the counting of equal groups, the calculation of the area of a rectangle, or the description of a rate [three times as many])
- That multiplication has those same meanings regardless of what number 3 is multiplying
- That multiplication can be accomplished in parts (the distributive principle), for example, $3 \times 1.5 = 3 \times 1 + 3 \times 0.5$

Although the planned lesson is likely to depend on the fact that students understand that 1.5 is 15 tenths or 1 and 5 tenths, a teacher could effectively teach the same lesson even to students who do not have that understanding or who simply are not ready to deal with decimals. The teacher could allow the less developed students to explore the concepts using whole numbers while the more advanced students are using decimals. Only when the teacher felt that the use of decimals was in an individual student's zone of proximal development would the teacher ask that student to work with decimals. Thus, by making this adjustment, the teacher differentiates the task to locate it within each student's zone of proximal development.

ASSESSING STUDENTS' NEEDS

For a teacher to teach to a student's zone of proximal development, first the teacher must determine what that zone is by gathering diagnostic information to assess the student's mathematical developmental level. For example, to determine a 3rd- or 4th-grade student's developmental level in multiplication, a teacher might use a set of questions to find out whether the student knows various meanings of multiplication, knows to which situations multiplication applies, can solve simple problems involving multiplication, and can multiply single-digit numbers, using either memorized facts or strategies that relate known facts to unknown facts (e.g., knowing that 6×7 must be 7 more than 5×7).

Some tools to accomplish this sort of evaluation are tied to developmental continua that have been established to describe students' mathematical growth (Small, 2005a, 2005b, 2006, 2007, 2010b; Small et al., 2011a, 2011b). Teachers might also use locally or personally developed diagnostic tools. Only after a teacher has determined a student's level of mathematical sophistication, can he or she even begin to attempt to address that student's needs.

PRINCIPLES AND APPROACHES TO DIFFERENTIATING INSTRUCTION

Differentiating instruction is not a new idea, but the issue has been gaining an ever higher profile for mathematics teachers in recent years. More and more, educational systems and parents are expecting the teacher to be aware of what each individual student needs and to plan instruction to focus on those needs. In the past, this was less the case in mathematics than in other subject areas, but now the expectation is common in mathematics as well.

There is general agreement that to effectively differentiate instruction, the following elements are needed:

- ***Big Ideas.*** The focus of instruction must be on the **big ideas** being taught to ensure that they all are addressed, no matter at what level.
- ***Choice.*** There must be some aspect of choice for the student, whether in content, process, or product.

- ***Preassessment.*** Prior assessment is essential to determine what needs different students have (Gregory & Chapman, 2006; Murray & Jorgensen, 2007).

Teaching to Big Ideas

The Curriculum Principle of the NCTM *Principles and Standards for School Mathematics* states that "A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades" (NCTM, 2000, p. 14). The introduction to the Common Core State Standards indicates that the Standards not only stress conceptual understanding of key ideas but also continually return to organizing principles, such as properties of operations, to structure those ideas (Common Core State Standards Initiative, 2010).

Curriculum coherence requires a focus on interconnections, or big ideas. Big ideas represent fundamental principles; they are the ideas that link the specifics. For example, the notion that **benchmark numbers** are a way to make sense of other numbers is equally useful for the 1st-grader who relates the number 8 to the more familiar 10, the 4rd-grader who relates $\frac{3}{8}$ to the more familiar $\frac{1}{2}$, or the 7th-grader who relates π to the number 3. If students in a classroom differ in their readiness, it is usually in terms of the specifics and not the big ideas. Although some students in a classroom where rounding of decimal thousandths to appropriate benchmarks is being taught might not be ready for that precise topic, they could still deal with the concept of estimating, when it is appropriate, and why it is useful.

Big ideas can form a framework for thinking about "important mathematics" and supporting standards-driven instruction. Big ideas find application across all grade bands. There may be differences in the complexity of their application, but the big ideas remain constant. Many teachers believe that curriculum requirements limit them to fairly narrow learning goals and feel that they must focus instruction on meeting those specific student outcomes. Differentiation requires a different approach, one that is facilitated by teaching to the big ideas.

Choice

Many math teachers are still not comfortable with the notion of student choice except in the rarest of circumstances. They worry that students will not make "appropriate" choices.

However, some teachers who are uncomfortable differentiating instruction in terms of the main lesson goal are willing to provide some choice in follow-up activities students use to practice the ideas they have been taught. Some of the strategies that have been suggested for differentiating practice include use of menus from which students choose, tiered lessons in which teachers teach to the whole group and vary the follow-up for different students, learning stations where different students attempt different tasks, or other approaches that allow for student choice, usually in pursuit of the same basic overall lesson goal (Tomlinson, 1999; Westphal, 2007).