



# Contents

---

<b>Foreword, by Diane Heacox</b>	v
<b>Preface</b>	vii
Organization of the Book	vii
Organization of the Content Chapters	ix
Changes in the Second Edition	ix
<b>Acknowledgments</b>	xiii
<b>1 Why and How to Differentiate Math Instruction</b>	<b>1</b>
The Challenge in Math Classrooms	1
What It Means to Meet Student Needs	3
Assessing Students' Needs	4
Principles and Approaches to Differentiating Instruction	4
Two Core Strategies for Differentiating Mathematics Instruction:	
Open Questions and Parallel Tasks	6
Creating a Math Talk Community	14
<b>2 Number and Operations</b>	<b>15</b>
Topics	15
The Big Ideas for Number and Operations	16
Open Questions for Prekindergarten–Grade 2	17
Open Questions for Grades 3–5	29
Open Questions for Grades 6–8	40
Parallel Tasks for Prekindergarten–Grade 2	48
Parallel Tasks for Grades 3–5	55
Parallel Tasks for Grades 6–8	65
Summing Up	72
<b>3 Geometry</b>	<b>73</b>
Topics	73
The Big Ideas for Geometry	74
Open Questions for Prekindergarten–Grade 2	75
Open Questions for Grades 3–5	81
Open Questions for Grades 6–8	89
Parallel Tasks for Prekindergarten–Grade 2	97
Parallel Tasks for Grades 3–5	101
Parallel Tasks for Grades 6–8	106
Summing Up	110

<b>4 Measurement</b>	<b>111</b>
Topics	111
The Big Ideas for Measurement	112
Open Questions for Prekindergarten–Grade 2	113
Open Questions for Grades 3–5	117
Open Questions for Grades 6–8	123
Parallel Tasks for Prekindergarten–Grade 2	126
Parallel Tasks for Grades 3–5	130
Parallel Tasks for Grades 6–8	135
Summing Up	140
<b>5 Algebra</b>	<b>141</b>
Topics	141
The Big Ideas for Algebra	142
Open Questions for Prekindergarten–Grade 2	143
Open Questions for Grades 3–5	146
Open Questions for Grades 6–8	151
Parallel Tasks for Prekindergarten–Grade 2	156
Parallel Tasks for Grades 3–5	158
Parallel Tasks for Grades 6–8	161
Summing Up	164
<b>6 Data Analysis and Probability</b>	<b>165</b>
Topics	165
The Big Ideas for Data Analysis and Probability	166
Open Questions for Prekindergarten–Grade 2	167
Open Questions for Grades 3–5	172
Open Questions for Grades 6–8	175
Parallel Tasks for Prekindergarten–Grade 2	183
Parallel Tasks for Grades 3–5	186
Parallel Tasks for Grades 6–8	189
Summing Up	198
<b>Conclusions</b>	<b>199</b>
The Need for Manageable Strategies	199
Developing Open Questions and Parallel Tasks	200
The Benefits of These Strategies	202
<b>Appendix: Worksheet for Open Questions and Parallel Tasks</b>	<b>203</b>
<b>Glossary</b>	<b>205</b>
<b>Bibliography</b>	<b>215</b>
<b>Index</b>	<b>219</b>
Index of Subjects and Cited Authors	219
Index of Big Ideas	221
<b>About the Author</b>	<b>223</b>



## Foreword

---

**MARIAN SMALL'S** second edition of *Good Questions: Great Ways to Differentiate Mathematics Instruction* will be greatly celebrated by this educator. In my work both in university courses and in professional development with teachers, I have faced specific questions about how to differentiate mathematics. Until the first edition of *Good Questions* was published, I did not have a resource to suggest that was solidly built on research and best practices in mathematics education and that provided strategies that were both doable and practical for classroom teachers. I have shared the book with mathematics teachers across the United States and Canada, and all agree that it is solidly built on best practices in mathematics and is an amazing resource for teachers.

The strategies in this book enable teachers to adjust mathematics instruction to meet the needs of their students who are at different developmental levels. This is a critical element in differentiating instruction for all students and increases the likelihood that they will be successful in learning.

The book, while honoring and utilizing the wisdom of the National Council of Teachers of Mathematics (NCTM) and framing the text around NCTM's five content strands, is correlated with the Common Core State Standards (CCSS). What has been particularly helpful for teachers is the organization of each chapter. Strategies are organized by content strand and grade level, so it is incredibly easy for teachers to identify the content strands and respective strategies for their grade level.

Marian Small supports the underlying principles and approaches to differentiating instruction by emphasizing the significance of preassessment and encouraging teachers to frame their instruction around big ideas. Her core strategies enable teachers to clearly respond to learning differences in mathematics and match students with "just right" learning experiences. In addition, she recognizes the impact of interest on student motivation by modeling appropriate methods for utilizing student choice in differentiating mathematics.

Small focuses on two core strategies for differentiating mathematics instruction. These core strategies are open questions and parallel tasks. Open questions are framed in such a way that all students can respond, regardless of their stage of mathematical development. She thoroughly explains the process for designing such questions and provides ample examples across content strands and grade levels. Teachers can go directly to the book to select among the sample questions for immediate use in their discussions! Although Small is using this core strategy

for mathematics, I have used her six design strategies for creating open questions in every curriculum area. Using her process ensures that teachers design questions that truly engage students in the 21st-century skills of critical and creative thinking.

Parallel tasks are Small's version of tiered assignments. Although tiered assignments can be designed in a variety of ways for a variety of purposes, Small constructs her mathematics parallel tasks around developmental levels. Students are all working on the same big idea within an NCTM strand, but they are engaged in tasks that reflect different levels of complexity. From my experience, effectively designing tiered assignments in mathematics demands deep mathematical understanding. Again, Small provides ample ready-to-go parallel tasks across content strands and grade levels. Even less confident or novice mathematics educators can let these ideas work for them!

Common Core State Standards are noted with each example of an open question or parallel task. This is a great benefit for teachers working with these standards. In addition, a template is provided to assist teachers in designing their own open questions and parallel tasks.

A helpful glossary defines technical terms used throughout the book. And a bibliography highlights excellent resources for teachers who want to go farther or deeper in differentiating mathematics.

This book will be a well-used resource and reference for teachers striving to differentiate mathematics instruction. Marian Small's *Good Questions: Great Ways to Differentiate Mathematics Instruction* is founded on mathematics research and best practices and provides both practical and doable strategies that enable teachers to confidently differentiate mathematics.

—Diane Heacox



## Preface

---

**IN 2001**, I embarked on a long-term research study to investigate how mathematical understanding develops in students through the elementary years. The project was housed at the University of New Brunswick and the University of Toronto, and it was supported by Nelson Education Ltd. The work ultimately led to a professional development program called PRIME, which explores student development in mathematics. The research involved testing over 12,000 kindergarten through 7th-grade students.

The research resulted in the creation of developmental maps that show the stages students move through in each of the strands of mathematics articulated by the National Council of Teachers of Mathematics (Small, 2005a, 2005b, 2006, 2007, 2010).

The value of the maps is in helping teachers adjust instruction to meet the needs of students who are at different levels of development. To that end, I developed two strategies that are relatively easy to implement and that allow teachers to make such curriculum adjustments. These strategies are open questions and parallel tasks.

### ORGANIZATION OF THE BOOK

An introductory chapter describes the rationale for differentiating math instruction and explains the two principal strategies that are employed throughout the book: open questions and parallel tasks. Five content chapters then illustrate application of these strategies, followed by a final summary chapter, an appendix containing a template for teachers wishing to develop their own materials, a glossary, a bibliography, and an index.

Chapters 2–6 focus on the five content strands enunciated in the *Principles and Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM):

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability (NCTM, 2000)

The content strands are not developed sequentially, and Chapters 2–6 can be approached in any order.

There was some consideration of organizing the book according to the headings of the Common Core State Standards, but I decided to remain with the more familiar and more general organization according to the NCTM Standards to match the previous edition. Specific correlations with the Common Core State Standards are indicated within the text.

Mathematical concepts are addressed in a framework of big ideas, which have been developed by marrying the NCTM process standards of problem solving, reasoning and proof, communicating, connecting, and representing (NCTM, 2000) with the NCTM content standards for the five content strands listed above. Big ideas are statements of fundamental principles and are broadly applicable to multiple grade bands and different developmental levels of understanding.

Within each of the content chapters, the suggested differentiating questions and tasks are divided according to the primary, elementary, and middle-level grade bands set out in the NCTM (2000) standards:

- Prekindergarten–grade 2
- Grades 3–5
- Grades 6–8

The choice of grade band for a particular question reflects the new Common Core State Standards.

The object of differentiation is to teach the same broad concepts to students at different developmental levels. The multilayered organization of material within Chapters 2–6 is intended to help make this possible.

The Appendix features a template worksheet that will assist teachers in developing their own materials in support of differentiated instruction through use of open questions and parallel tasks. An example of application of the worksheet appears in Chapter 1.

The Glossary defines technical terms used throughout. Each word that appears in the Glossary is shown in boldface type at its first occurrence in the text, and each Glossary entry is annotated with the chapter and page number of the term's first occurrence.

The Bibliography highlights three types of resources: those that are referenced in other parts of the text, those that speak to the issues of teaching to big ideas and differentiating instruction, as well as a number of excellent sources for activities that can be used as-is or used as a starting point for creating open questions and parallel tasks.

The Index focuses on educational concepts—standards, student development, teaching methods and principles, and such—as opposed to mathematical concepts. To facilitate user access to the mathematical topics covered, an Index of Big Ideas is provided, listing all big ideas covered in the content chapters.

## ORGANIZATION OF THE CONTENT CHAPTERS

Chapters 2–6 address the five NCTM content strands, providing examples of open questions and parallel tasks—organized around big ideas—for the prekindergarten–grade 2, grades 3–5, and grades 6–8 grade bands.

Each chapter begins with a listing of the goals of the NCTM standard for the particular content strand, followed by a brief description of how student understanding of the content develops across grade levels from prekindergarten through grade 8. For each grade band, concepts students will learn and apply are described, demonstrating how basic concepts form the foundation for more complex concepts as understanding develops. The content standards are approached through exploration of big ideas, which are listed at the beginning of each chapter.

The bulk of each chapter is composed of a series of illustrations of application of the two differentiation strategies that are the focus of this book: open questions and parallel tasks. Each of these strategies is discussed for each of the three grade bands. Within each grade band section, content is organized by big idea, often with several illustrations for each big idea. Readers may choose to focus on the grade band in which they teach, or they may choose to study all of the grade bands to get a better sense of how understanding of the content develops in students.

For many of the questions and tasks described, important follow-up questions that can be used in the classroom are listed. In many cases, variations of activities or follow-up questions are provided to allow for even more flexibility in differentiating instruction. In addition, the rationale for each activity is presented as background that will make it easier for teachers to develop their own related activities.

Numerous Teaching Tips are included in each chapter. These sometimes relate to a specific activity, but often they are general strategies that can apply to any situation in which a teacher is attempting to differentiate instruction in math.

At the end of each chapter, concluding remarks include a few suggestions for developing additional open questions and parallel tasks by using the template provided in the Appendix.

## CHANGES IN THE SECOND EDITION

Readers familiar with the first edition of this book will note that there are many new questions. In addition, there has been some rewording of the big ideas that are used to frame each chapter.

A more significant change involves addressing the Common Core State Standards (CCSS), which has led to moving some questions from one grade band to another and adding notations relating each question to a domain heading of the Common Core State Standards.

CCSS content designations are referenced in text sections as indicated in the chart on the next page.

COMMON CORE STATE STANDARDS FOR MATHEMATICS		
Mathematical Content Domains	Grade Band	Chapter(s)
Counting and Cardinality	Pre-K–Grade 2	2
Operations and Algebraic Thinking	Pre-K–Grade 2; Grades 3–5	2 and 5
Number and Operations in Base Ten	Pre-K–Grade 2; Grades 3–5	2
Measurement and Data	Pre-K–Grade 2; Grades 3–5	4 and 6
Geometry	Pre-K–Grade 2; Grades 3–5; Grades 6–8	3
Number and Operations—Fractions	Grades 3–5	2
Ratios and Proportional Relationships	Grades 6–8	2
The Number System	Grades 6–8	2
Expressions and Equations	Grades 6–8	5
Statistics and Probability	Grades 6–8	6
Functions	Grades 6–8	5
<b>Mathematical Practices:</b> All Grade Bands, Chapters 2–6		
<ol style="list-style-type: none"> <li>1. Making sense of problems and persevering in solving them</li> <li>2. Reasoning abstractly and quantitatively</li> <li>3. Constructing viable arguments and critiquing the reasoning of others</li> <li>4. Modeling with mathematics</li> <li>5. Using appropriate tools strategically</li> <li>6. Attending to precision</li> <li>7. Looking for and making use of structure</li> <li>8. Looking for and expressing regularity in repeating reasoning</li> </ol>		

In addition to the standards that describe content, there are eight Common Core State Standards that focus on mathematical practice, a number of which regularly underlie the open questions or parallel tasks in this resource. The eight practices are listed in the chart above.

These practice standards work hand in hand with the CCSS content domain standards. They reflect “processes” that should be elicited from students and goals for orienting and developing students as mathematical thinkers as various content is learned.

Some of these practices may or may not be brought out, depending on how a teacher handles the presented questions or tasks, but others are implicit no matter what direction the discussion of the question or task takes.



For example, almost all of the questions and tasks posed in this book require students to make sense of them and persevere in solving them. This is true with open questions, which are often deliberately vague and require students to make sense of the question before they can choose the direction in which to go. This is also true with parallel tasks, where students must make sense of both options to decide with which one to proceed.

Students are frequently asked to explain their rationale, requiring them to construct viable arguments. Ideally, teachers would encourage students to critique each other's reasoning, although that would not be implicit in the question itself.

A great many of the questions provided require students to reason, either abstractly or, in the case of number or measurement questions, quantitatively.

Some questions bring out the other practices: modeling, using appropriate tools strategically, attending to precision, looking for and making use of structure, and/or looking for and expressing regularity in repeated reasoning. There are fewer questions attending to precision in this particular resource than in others precisely because differentiation must allow for varying levels of precision.

A focus on expressing regularity in repeated reasoning is also less prominent in this particular resource because the questions are frequently intended to generate initial discussion rather than to tie down rules. There are a few examples, however, that do have that focus.

Listed with each question and task are those common core mathematical practices that are clearly evoked in the question or task. This does not mean that teachers might not also evoke others, depending on their approach to student responses or the student responses themselves.

**IT IS MY HOPE** that teachers will embrace the two core strategies—open questions and parallel tasks—that are explained and demonstrated in this book, and find them, as I have, to be helpful to the many children who come into classrooms with highly differentiated mathematical preparation, skill, and confidence. Seeing a child who has been struggling in mathematics start to feel successful is an important goal in my teaching. I have seen the use of the strategies described in this volume make that happen over and over again.



# Why and How to Differentiate Math Instruction

---

**STUDENTS IN ANY CLASSROOM** differ in many ways, only some of which the teacher can reasonably attend to in developing instructional plans. Some differences will be cognitive—for example, what previous concepts and skills students can call upon. Some will be more about learning style and preferences, including behaviors such as persistence or inquisitiveness or the lack thereof; whether the student learns better through auditory, visual, or kinesthetic approaches; and personal interests.

## THE CHALLENGE IN MATH CLASSROOMS

Although many teachers of language arts recognize that different students need different reading material, depending on their reading level, it is much less likely that teachers vary the material they ask their students to work with in mathematics. The math teacher will more frequently teach all students based on a fairly narrow curriculum goal presented in a textbook. The teacher will recognize that some students need additional help and will provide as much support as possible to those students while the other students are working independently. Perhaps this occurs because **differentiating instruction** in mathematics is a relatively new idea. Perhaps it is because it is not easy in mathematics to simply provide an alternate book to read (as can be done in language arts). And perhaps it is because teachers may never have been trained to really understand how students differ mathematically. However, students in the same grade level clearly *do* differ mathematically in significant ways. Teachers want to be successful in their instruction of all students, and feel even more pressure to do so in the current social climate. Understanding differences and differentiating instruction are important processes for achievement of that goal.

The National Council of Teachers of Mathematics (NCTM), the professional organization whose mission it is to promote, articulate, and support the best possible teaching and learning in mathematics, recognizes the need for differentiation. The first principle of the NCTM *Principles and Standards for School Mathematics* reads, “Excellence in mathematics education requires equity—high expectations and strong support for all students” (NCTM, 2000, p. 12).

In particular, NCTM recognizes the need for accommodating differences among students, taking into account prior knowledge and intellectual strengths, to ensure that each student can learn important mathematics. “Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students” (NCTM, 2000, p. 12).

## How Students Might Differ

One way that we see the differences in students is through their responses to the mathematical questions and problems that are put to them. For example, consider the task below, which might be asked of 3rd-grade students:

In one cupboard, you have three shelves with five boxes on each shelf. There are three of those cupboards in the room. How many boxes are stored in all three cupboards?

Students might approach the task in very different ways. Here are some examples:

- Liam immediately raises his hand and simply waits for the teacher to help him.
- Angelita draws a picture of the cupboards, the shelves, and the boxes and counts each box.
- Tara uses addition and writes  $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$ .
- Dejohn uses addition and writes  $5 + 5 + 5 = 15$ , then adds again, writing  $15 + 15 + 15 = 45$ .
- Rebecca uses a combination of multiplication and addition and writes  $3 \times 5 = 15$ , then  $15 + 15 + 15 = 45$ .

## The Teacher’s Response

What do all these different student approaches mean for the teacher? They demonstrate that quite different forms of feedback from the teacher are needed to support the individual students. For example, the teacher might wish to:

- Follow up with Tara and Dejohn by introducing the benefits of using a multiplication expression to record their thinking.
- Help Rebecca extend what she already knows about multiplication to more situations.
- Encourage Liam to be more independent, or set out a problem that is more suitable to his developmental level.
- Open Angelita up to the value of using more sophisticated strategies by setting out a problem in which counting becomes even more cumbersome.

These differences in student approaches and appropriate feedback underscore the need for a teacher to know where his or her students are developmentally to be able to meet each one's educational needs. The goal is to remove barriers to learning while still challenging each student to take risks and responsibility for learning (Karp & Howell, 2004).

## WHAT IT MEANS TO MEET STUDENT NEEDS

One approach to meeting each student's needs is to provide tasks within each student's **zone of proximal development** and to ensure that each student in the class has the opportunity to make a meaningful contribution to the class community of learners. Zone of proximal development is a term used to describe the "distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86).

Instruction within the zone of proximal development allows students, whether through guidance from the teacher or through working with other students, to access new ideas that are close enough to what they already know to make the access feasible. Teachers are not using educational time optimally if they either are teaching beyond a student's zone of proximal development or are providing instruction on material the student already can handle independently. Although other students in the classroom may be progressing, the student operating outside his or her zone of proximal development is often not benefiting from the instruction.

For example, a teacher might be planning a lesson on multiplying a decimal by a whole number. Although the skill that is the goal of the lesson is to perform a computation such as  $3 \times 1.5$ , there are three underlying mathematical concepts that a teacher would want to ensure that students understand. Students working on this question should know:

- What multiplication means (whether repeated addition, the counting of equal groups, the calculation of the area of a rectangle, or the description of a rate [three times as many])
- That multiplication has those same meanings regardless of what number 3 is multiplying
- That multiplication can be accomplished in parts (the distributive principle), for example,  $3 \times 1.5 = 3 \times 1 + 3 \times 0.5$

Although the planned lesson is likely to depend on the fact that students understand that 1.5 is 15 tenths or 1 and 5 tenths, a teacher could effectively teach the same lesson even to students who do not have that understanding or who simply are not ready to deal with decimals. The teacher could allow the less developed students to explore the concepts using whole numbers while the more advanced students are using decimals. Only when the teacher felt that the use of decimals was in an individual student's zone of proximal development would the teacher ask that student to work with decimals. Thus, by making this adjustment, the teacher differentiates the task to locate it within each student's zone of proximal development.