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◆ CHAPTER 1 ◆

Pedagogical Principles

Children Playing, Talking, Touching and Thinking Their Way to Understanding

THE IDEAS in this resource are based on the premise that there are a number of established pedagogical principles that must be respected in developing mathematical learning opportunities for young children.

These include the following:

- The importance of worthwhile mathematical tasks.
- The value of rich mathematical conversation.
- The value of teaching through play.
- The value of learning with concrete objects.
- The importance of differentiation and appropriate challenge in the classroom.
- The importance of listening to the child.

WHAT MAKES A TASK WORTHWHILE?

Different individuals will have different opinions about what makes a mathematical task worthwhile (Breyfogle & Williams, 2008). Many believe, though, that the following are critical components of a worthwhile task:

- The maths being addressed should be “substantial”; usually the tasks involve reasoning and connections.
- The task should not be too easy or too hard, but should present an appropriate challenge for the student.
- The task should be engaging.
- The task should not be overly scaffolded.
- The task has natural extensions.
- There are many ways to approach the task.

Some might suggest that the task be rooted in real-life situations (Wolf, 2015). I would argue that this depends on what is meant by “real-life situations”. I believe it is as real-life for students to play with pattern blocks to make a design as it is to purchase an item in a pretend shop.

Others might argue that the task should involve flexible use of technology (Learn Alberta, 2007). I feel that this can be useful but is not essential.

When a teacher chooses a task, they should ensure that the bulk of the time that students spend pursuing the task is spent thinking mathematically and not simply on “busy work” with a little bit of mathematical thinking thrown in.

Bay-Williams and Van de Walle (2010) presented a list of criteria for evaluating the worth of a task. They suggested that teachers rate a task on a scale of 1 to 4 based on attributes including whether the maths covered is significant, whether is it appropriate for the child, how engaging the task is and whether the task calls for true problem-solving.

Below is an alternate scale you might consider in judging whether a task is worthwhile for young students.

Goal: The task ...	Rating			
	1	2	3	4
is mathematically valuable.				
will extend students' mathematical knowledge.				
allows entry for all students.				
extends strong students.				
involves students in problem-solving/critical thinking/ creative thinking.				
engages students.				
supports collaboration.				
provides opportunity for rich maths conversation.				

For any teaching activity a teacher might consider, they might come up with a “worthwhile” score using the 1–4 rating scale, where 1 shows no evidence that the goal is likely to be met and 4 indicates that the goal is likely to be fully achieved with the task.

RICH MATHEMATICAL CONVERSATION

Most of us learn best when learning with others. Most of the tasks suggested in this resource are meant as learning opportunities rather than as assessment opportunities, so it makes sense that students engage in these activities in pairs or small groups. This also makes it more fun for the students.

Ideally, students working together are likely to engage in conversations that are less about one student telling another what to do and more about considering alternative approaches for making mathematical sense of the task. The focus is less on the answer and more on the mathematical thinking in which students engage (Hufferd-Ackles, Fuson & Sherin, 2004).

As well, the conversation with the teacher needs to be rich, and more than just questions from teachers that only require short answers from students.

THE VALUE OF TEACHING THROUGH PLAY

Early childhood educators have long advocated play-based learning; the belief is that an inquiry stance is crucial to early literacy and numeracy development (National Council for Curriculum and Assessment [NCCA], 2009). This is based, in large part, on students' innate curiosity about their world.

Many opportunities for mathematics arise during play. For example, while building, parents or educators might raise issues of counting (how many blocks were used), stability (which sorts of blocks were on the bottom versus the top), measurement (how tall or wide the structure is) and so forth. While playing with a tea set, parents or educators might raise issues of measurement, for example, how much small cups hold versus larger cups, or how many cups a teapot could actually fill. While playing dress-up, parents or educators might raise issues of measurement, in terms of sizes of clothing items.

Mathematics standards for young children are more likely to be met when a teacher ensures that appropriate materials are available to stimulate the child mathematically and then follows up the students' conversation in play with appropriate questions and challenges.

For example, if a child is kicking a ball, a parent or educator might ask how we could measure how far it was kicked. If a child is flying a kite, parents or educators could ask how the child could figure out if it went higher the last time or this time. If a child is hanging onto a high bar on a playground, a parent or educator could ask how the child will know how long they have been holding on.

Many of the maths games in this book are unnamed, but teachers might invite students to name some of the class favourites to make the games feel more like their own and to give them some ownership of mathematical ideas.

THE VALUE OF LEARNING USING CONCRETE OBJECTS

Mathematics is a very abstract subject. It is through looking at concepts concretely, with concrete materials, that students can make sense of mathematical concepts and move from the concrete to the abstract (National Research Council, 2001).

◆ CHAPTER 2 ◆

Counting and Cardinality

THIS CHAPTER focuses on the early experiences students should have as they learn to count a quantity to determine its numerical size, or **cardinality**.

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THE FUNDAMENTALS

Rote Versus Meaningful Counting

There is a big difference between a child who is able to say the counting sequence 1, 2, 3, 4, 5, ... and the child who can use that sequence to figure out how many objects are in a set. The former skill is called rote counting; the latter is more meaningful counting.

◆ CHAPTER 3 ◆

Operations and Algebraic Thinking

THIS CHAPTER focuses on what mathematical operations mean, how those meanings affect how we figure out sums and differences, and how we can make it engaging for young children to gain those understandings and connections.

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◆ CHAPTER 4 ◆

Number and Operations in Base Ten

THIS CHAPTER focuses on ideas related to place value and operations that are built on place value concepts.

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◆ CHAPTER 5 ◆

Measurement and Data

THIS CHAPTER focuses separately on both measurement and data concepts young students encounter.

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Geometry

THIS CHAPTER focuses on geometric and spatial concepts young students explore in preschool to Year 2.

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THE FUNDAMENTALS

The field of geometry includes the study of shapes (1-dimensional, 2-dimensional and 3-dimensional) as well as **spatial relationships**, which encompass positional relationships and feature relationships among shapes.