

INTRODUCTION

What is the **FOCUS** series?

FOCUS is a mathematics-strategy practice series. Each student book in the series provides brief instruction and concentrated practice for students in one targeted Mathematics Strategy. **FOCUS** also allows students the opportunity for self-assessment of their performance. It allows teachers the opportunity to identify and assess a student's level of mastery.

Six Mathematics Strategies featured in the **FOCUS** series:

- Building Number Sense
- Using Estimation
- Using Algebra
- Using Geometry
- Determining Probability and Averages
- Interpreting Graphs and Charts

The **FOCUS** series spans eight year levels, from year one to year eight. The introductory passages in each lesson are written at or below year level, allowing students to focus on the mathematics without struggling with the reading.

Book	Reading Level
Book A	at or below year one readability
Book B	at or below year two readability
Book C	at or below year three readability
Book D	at or below year four readability
Book E	at or below year five readability
Book F	at or below year six readability
Book G	at or below year seven readability
Book H	at or below year eight readability

What is Determining Probability and Averages, the Mathematics Strategy featured in this **FOCUS** book?

Determining probability involves finding the likelihood that an event will occur. Probability is determined by comparing a specific outcome with all of the possible outcomes in a given situation. In the early years, students study probability in terms of *more likely*, *less likely* and *equally likely* outcomes. As they progress, students learn to express probability in fraction form. The numerator represents a specific outcome, and the denominator represents the total number of possible outcomes. Students in the upper year levels also learn to express probability in percentage and decimal form.

Finding the average of a group of numbers provides information about how each number relates to the group as a whole. Students in years two to eight learn how to calculate averages. The addends that they work with increase in size and number as students progress through the year levels. In years seven and eight, students are introduced to several measures of central tendency. They learn to identify and calculate the mean, median, mode and range of a data set.

Students in year one learn important readiness concepts. They sort objects into groups. They practise sorting items by size, shape and colour. Students also combine sets into one group and then make equal groups.

In the middle years students learn to calculate the total number of possible combinations in a given situation. The number of possible combinations is determined by calculating the product of the numbers of items in the given categories.

How should I use the **FOCUS** series in the classroom?

The **FOCUS** series can be used effectively in the classroom in several ways. Here is a suggestion for using the program in **whole class, large group, small group, paired** and **individual** formats.

To the Student

(inside front cover of the student book)

Read and discuss this with the whole class or large group to make sure students understand what they are to do in the book.

Learn About

(pages 2–3 of the student book)

Read the two pages of instruction in the Mathematics Strategy to the whole class or large group. Model using the Mathematics Strategy. Use information from the Mathematics Strategy Tips for the Teacher on pages 12–13 of this teacher guide to prompt additional in-depth discussion of the Mathematics Strategy, as appropriate. Make sure all students understand the features of the Mathematics Strategy and how to apply the Mathematics Strategy before they go on. The Learn About requires approximately 45 minutes.

Lesson Preview

(pages 4–5 of the student book)

Read the boxed directions to the whole class or large group. Emphasise what students should watch for as they read the problem. Have students read the problem individually. Guide the whole class or large group in answering the two selected-response questions. Then discuss why each answer choice is correct or not correct. Make sure all students understand how to answer the Mathematics Strategy questions before they go on. The Lesson Preview requires approximately 45 minutes.

Lessons

(pages 6–45 of the student book)

For each lesson, have students read the directions and the passage individually, in pairs or in small groups. Have students answer the selected-response questions and the constructed-response question individually, in pairs or in small groups.

Have students use the Tracking Chart on page 47 of the student book to note the date that they have finished each lesson. When the questions in all five lessons in a group have been corrected, have students note the number of correct responses for each lesson and then the number of correct responses for the whole group of lessons.

Each lesson, plus tracking, requires approximately 45 minutes. Allow students 30 minutes to read the passage and answer the questions, and allow 15 minutes to discuss the responses. Discuss the answers to the questions with the whole class or large group, or with pairs, small groups or individuals. (See **What is the correction procedure?** on page 4 of this teacher guide.)

Self-Assessment: When students have finished each group of five lessons, have them complete the appropriate Self-Assessment. When students have finished all twenty lessons, have them complete Self-Assessment 5. Each Self-Assessment requires approximately 20 minutes.

Discussion: When students have finished each group of five lessons, discuss their performance individually or in small groups. When students have finished all twenty lessons, discuss their performance individually or in small groups. Each discussion requires approximately 25 minutes.

MATHEMATICS STRATEGY TIPS FOR THE TEACHER

Understanding probability can help you determine the chance that a certain event will occur.

Probability is the chance that a certain event will occur. It is calculated by dividing the number of favourable outcomes by the total number of possible outcomes. In the example at the top of the Learn About page, the number of favourable outcomes is the total number of cards of the specified colours. For this problem, there are two colours specified—blue *or* orange. Since either colour is a favourable outcome, the sum of blue cards and yellow cards (14) is the number of favourable outcomes. The number of possible outcomes is the total number of cards in the bag (56). The probability of picking a blue card or an orange card is $\frac{14}{56}$, or $\frac{1}{4}$.

Stress to students the importance of the word *or* in this problem. It indicates that more than one outcome is favourable. Reinforce this concept by having students calculate the probability of picking a red card *or* a black card given the data in the chart ($\frac{10}{56} + \frac{10}{56} = \frac{20}{56} = \frac{5}{14}$).

Probability can be expressed as a fraction, as a decimal or as a percentage. In fraction form, the numerator is the number of favourable events, and the denominator is the total number of possible events.

A common error when expressing probability as a fraction is to place the total number of possible events in the numerator, and the number of favourable events in the denominator. Explain to students that when probability is expressed as a fraction, the denominator is almost always greater than the numerator.

Learn About

Determining Probability and Averages: Probability

Probability is the chance that a certain event will occur. The probability of an event occurring is found by dividing the number of favourable outcomes by the number of possible outcomes. Probability can be represented as a fraction or as a per cent.

A bag contains several cards. Each card is the same size and shape. The number of cards of each colour is shown in the chart.

To find the probability of picking a blue card or an orange card, first find the total number of cards. Then divide the number of cards that are blue or orange by the total number of cards. You can write the probability as a fraction or as a per cent.

Cards	
Colour	Number
Red	10
Green	15
Blue	8
Black	10
Yellow	7
Orange	6

Fraction:

Number of cards: 56

Total number of cards that are blue or orange: 14

Probability: $\frac{14}{56} = \frac{1}{4}$

Per cent:

$\frac{1}{4} = 1 \div 4 = 0.25$

$0.25 \times 100 = 25\%$

Renée's PE teacher is selecting team captains for the volleyball teams. The teacher writes each student's name on a slip of paper and places the papers in a box. She then selects names from the box. The person whose name is selected first will be the captain of team A. There are 15 girls and 13 boys in Renée's class. What is the probability that a girl will be selected as the captain of team A?

There are 28 students in all and there are 15 girls.

The probability that a girl will be selected to be team A's captain is $\frac{15}{28}$.



Remember: Probability is the chance that a certain event will occur. The probability of an event occurring is found by dividing the number of favourable outcomes by the number of possible outcomes.

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Determining Probability and Averages Book H CAS0482 • © 2010 Hawker Brownlow Education

To express probability as a percentage, divide the numerator by the denominator. The quotient is a decimal form of the fraction. Multiply the decimal by 100 to express the decimal as a percentage.

Have students express the probability of picking a red card or a black card as a percentage ($\frac{5}{14} \approx 36\%$). Have students round the decimal to the nearest hundredth before converting it to a percentage.

Have students read the problem in the shaded box on the Learn About page. Ask them to identify the favourable outcomes and total possible outcomes. Have students describe how they would express this probability as a decimal and as a percentage.

How Is *FOCUS on Mathematics* Supported by Research?

FOCUS on Mathematics is supported by research from mathematical researchers and organisations, including the National Math Advisory Panel and National Council of Teachers of Mathematics, both from the US. Much of the research on effective instruction for mathematical students parallels the recommendations of the NMAP (2008). Many of these recommendations are integrated into the *FOCUS on Mathematics* series, including: word-problem focus, explicit instruction with modelling and focused practice.

Word-Problem Focus

Word problems are the proving ground for students to demonstrate their mastery of mathematical fluency and conceptual understanding. Having the ability to transfer what they have learned to new problem-solving situations is one of the major goals for mathematical education (NCTM, 2006; NMAP, 2008). “The issue of transfer, that is, the ability to use skills learned to solve one class of problems, such as similar triangles, to solve another class of problems, such as linear algebra, is a vital part of mathematics learning” (NMAP, 2008, p. 30). And yet, students, on average, have the most difficulty solving word problems.

The *FOCUS on Mathematics* series provides repeated and focused practice of key maths strategies in the context of word problems. With more than 800 word problems in the series, students gain multiple opportunities to practise core maths concepts and strategies.

Explicit Instruction with Modelling

Explicit instruction is a hallmark of effective instruction for struggling and on-level students. Explicit instruction is one of the instructional methods that research has proved to be effective. “By the term *explicit instruction*, it is meant that teachers provide clear models for solving a problem type using an array of examples, that students receive extensive practice in use of newly learned strategies and skills, that students are provided with opportunities to think aloud (i.e. talk through the decisions they make and the steps they take), and that students are provided with extensive feedback” (NMAP, 2008, p. 23). Each of these features, can be found in the *FOCUS on Mathematics* series. With explicit instruction and teacher modelling, skill efficiency is nearly guaranteed by students (Hiebert & Grouws, 2008).

FOCUS on Mathematics uses explicit instruction in the teaching of the mathematical strategies. The explicit instruction occurs in the Learn About section and the Lesson Preview section. Through the Learn About section, students receive explicit instruction consisting of a definition, semi-concrete and visual representations of the maths concepts, and a usage rule for the maths strategy. Additionally, *FOCUS on Mathematics* is a perfect vehicle for struggling students because it does not overwhelm students with the presentation of information. In the Learn About lesson, students initially experience the maths concepts in short presentations, usually three to seven sentences long. A Remember box text feature is a point of reference for students to use while attending to lessons. The Remember box is consistently placed in each book of the series. Struggling or novice maths students usually skip or gloss over text features, which are valuable tools. With repeated exposure and external prompting by the teacher, students learn to pay attention to the text feature.

Focused Practice

One of the major callings from from expert panels and organisations is for deeper learning and practice of mathematical skills and strategies. Focusing on specific key mathematical topics allows “teachers to commit more time each year to topics receiving special emphasis. At the same time, students would have opportunities to explore these topics in depth, in the context of related content and connected applications, thus developing more robust mathematical understandings” (NCTM, 2006, p. 4).

ANSWER KEY

FOCUS on Determining Probability and Averages, Book H

Lesson 1 (page 6)

1. C 2. A 3. C 4. B

5. Solution: The mean with the higher test score included is now 89.

Sample Explanation: *First, I found the sum of all the test scores, replacing the 75 from the original set of scores with 92.*

$$92 + 88 + 85 + 89 + 83 + 94 + 90 + 88 + 95 = 804$$

Then I divided the sum by the number of tests, which is 9.

$$804 \div 9 \approx 89$$

Lesson 2 (page 8)

1. A 2. B 3. C 4. A

5. Solution: The probability that the next student will choose a mystery or drama book is $\frac{31}{75}$.

Sample Explanation: *To determine the probability of choosing one item or another, I added the probabilities of choosing each item. I found the sum of $P(\text{mystery})$ and $P(\text{drama})$ to determine $P(\text{mystery or drama})$.*

$$P(\text{mystery}) = \frac{15}{75}$$

$$P(\text{drama}) = \frac{16}{75}$$

$$P(\text{mystery or drama}) = \frac{15}{75} + \frac{16}{75} = \frac{31}{75}$$

Lesson 3 (page 10)

1. C 2. B 3. D 4. B

5. Solution: Both the mode and the median changed to 87.

Sample Explanation: *I found the new mean, median, mode and range with the new mark factored in. The new mean was 87.6, which still rounds to 88.*

$$(94 + 83 + 89 + 76 + 97 + 87 + 87) \div 7 \\ 613 \div 7 \approx 88$$

The highest and lowest marks did not change, so the range did not change.

$$97 - 76 = 21$$

There had been no mode before and now the mode is 87, and there was a new number in the middle position, so the median also changed.

$$76, 83, 87, \underline{87}, 89, 94, 97$$

Lesson 4 (page 12)

1. C 2. B 3. A 4. D

5. Solution: The probability that the team will get a category that they want is $\frac{2}{5}$.

Sample Explanation: *I found the sum of the probabilities of landing on 3, 4 and "your choice" to find the probability of landing on any one of these spots.*

$$P(\text{category 3}) = \frac{2}{10}$$

$$P(\text{category 4}) = \frac{1}{10}$$

$$P(\text{your choice}) = \frac{1}{10}$$

$$\frac{2}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$$

Lesson 5 (page 14)

1. B 2. B 3. A 4. B

5. Solution: The range and the mode changed. Sample Explanation: *First, I replaced the ages of the two students who left the workshop with the ages of the two new students. Then I found the new mean, median, mode and range.*

$$\text{Mean: } (12 + 12 + 13 + 15 + 13 + 12 + 14 + 13 + 13 + 12 + 14 + 15) \div 12 \approx 13.2 \text{ or } 13$$

$$\text{Median: } 12, 12, 12, 12, 13, \underline{13, 13}, 13, 14, 14, 15, 15$$

$$\text{Range: } 15 - 12 = 3$$

$$\text{Mode: } 12 \text{ and } 13$$

The youngest age is now 12, so the new range is 3 years. There had been only one mode of 12 before, and now there are two modes, 12 and 13. In both cases, the rounded whole number for the mean was 13, so the mean did not change. The median did not change either.

Lesson 6 (page 16)

1. A 2. C 3. B 4. D

5. Solution: The mean song length on the CD, including the bonus song, is 4:01.

Sample Explanation: *First, I converted all song lengths to seconds and found the sum, including the bonus song. Then I divided the sum by 7.*

$$(228 \text{ sec} + 176 \text{ sec} + 207 \text{ sec} + 285 \text{ sec} + 228 \text{ sec} + 270 \text{ sec} + 292 \text{ sec}) \div 7$$

$$1686 \text{ sec} \div 7 \approx 240.86 \text{ sec}$$

Finally, I converted back to minutes and seconds.
 $240.86 \text{ sec} \div 60 \approx 4 \text{ minutes } 1 \text{ second}$