

INTRODUCTION

What is the *FOCUS* series?

FOCUS is a mathematics-strategy practice series. Each student book in the series provides brief instruction and concentrated practice for students in one targeted Mathematics Strategy. *FOCUS* also allows students the opportunity for self-assessment of their performance. It allows teachers the opportunity to identify and assess a student's level of mastery.

Six Mathematics Strategies featured in the *FOCUS* series:

- Building Number Sense
- Using Estimation
- Using Algebra
- Using Geometry
- Determining Probability and Averages
- Interpreting Graphs and Charts

The *FOCUS* series spans eight year levels, from year one to year eight. The introductory passages in each lesson are written at or below year level, allowing students to focus on the mathematics without struggling with the reading.

Book	Reading Level
Book A	at or below year one readability
Book B	at or below year two readability
Book C	at or below year three readability
Book D	at or below year four readability
Book E	at or below year five readability
Book F	at or below year six readability
Book G	at or below year seven readability
Book H	at or below year eight readability

What is Using Algebra, the Mathematics Strategy featured in this *FOCUS* book?

Algebra is the strand of mathematics that focuses on relationships among quantities and on the different ways to represent these relationships. One such representation uses numbers, symbols and variables. These elements can be combined to describe a mathematical situation. In the earlier years, students learn to use number sentences, which consist of numbers, symbols such as $+$, $-$, \times , \div , $=$ and variables that identify missing information. Variables are typically shown as blanks or boxes in years one and two. Beginning in year three, letter variables are introduced.

As they progress through the year levels, students learn about different kinds of representations of relationships. They write and solve equations, expressions and inequalities. Students learn to translate among different representations, such as equations, tables of values and graphs. These concepts lead to more comprehensive understanding of functions.

Patterns appear frequently in algebra problems. Some patterns appear as a list of sequentially ordered numbers. Other patterns appear as IN and OUT boxes or in tables. Students find missing elements and extend both number and geometric patterns. Patterns are one more way of representing relationships.

How should I use the **FOCUS** series in the classroom?

The **FOCUS** series can be used effectively in the classroom in several ways. Here is a suggestion for using the program in **whole class, large group, small group, paired** and **individual** formats.

To the Student

(inside front cover of the student book)

Read and discuss this with the whole class or large group to make sure students understand what they are to do in the book.

Learn About

(pages 2–3 of the student book)

Read the two pages of instruction in the Mathematics Strategy to the whole class or large group. Model using the Mathematics Strategy. Use information from the Mathematics Strategy Tips for the Teacher on pages 12–13 of this teacher guide to prompt additional in-depth discussion of the Mathematics Strategy, as appropriate. Make sure all students understand the features of the Mathematics Strategy and how to apply the Mathematics Strategy before they go on. The Learn About requires approximately 45 minutes.

Lesson Preview

(pages 4–5 of the student book)

Read the boxed directions to the whole class or large group. Emphasise what students should watch for as they read the problem. Have students read the problem individually. Guide the whole class or large group in answering the two selected-response questions. Then discuss why each answer choice is correct or not correct. Make sure all students understand how to answer the Mathematics Strategy questions before they go on. The Lesson Preview requires approximately 45 minutes.

Lessons

(pages 6–45 of the student book)

For each lesson, have students read the directions and the passage individually, in pairs or in small groups. Have students answer the selected-response questions and the constructed-response question individually, in pairs or in small groups.

Have students use the Tracking Chart on page 47 of the student book to note the date that they have finished each lesson. When the questions in all five lessons in a group have been corrected, have students note the number of correct responses for each lesson and then the number of correct responses for the whole group of lessons.

Each lesson, plus tracking, requires approximately 45 minutes. Allow students 30 minutes to read the passage and answer the questions, and allow 15 minutes to discuss the responses. Discuss the answers to the questions with the whole class or large group, or with pairs, small groups or individuals. (See **What is the correction procedure?** on page 4 of this teacher guide.)

Self-Assessment: When students have finished each group of five lessons, have them complete the appropriate Self-Assessment. When students have finished all twenty lessons, have them complete Self-Assessment 5. Each Self-Assessment requires approximately 20 minutes.

Discussion: When students have finished each group of five lessons, discuss their performance individually or in small groups. When students have finished all twenty lessons, discuss their performance individually or in small groups. Each discussion requires approximately 25 minutes.

MATHEMATICS STRATEGY TIPS FOR THE TEACHER

Some equations include percentages. To solve problems with percentages, you can use either the per cent equation or the proportion equation.

Both the per cent equation and the proportion equation have three parts – per cent, base and percentage. You may wish to provide several examples and ask students to name the base, per cent and percentage. Examples: 10 is 20% of 50 and 30% of 40 is 12.

Write these generic equations on a card for students to use as a reference when solving percentage problems.

Per cent equation:

$$\begin{array}{ccccccc} p\% & \text{of} & x & = & y \\ \uparrow & & \uparrow & & \uparrow \\ \text{per cent} & & \text{base} & & \text{percentage} \end{array}$$

per cent

$$\text{Proportion equation: } \frac{p}{100} = \frac{y}{x} \quad \leftarrow \text{percentage}$$

base

You can use the per cent equation to determine 15% of 30 by substituting the known values into the equation. Substitute 15% for p and 30 for x .

$$15\% \text{ of } 30 = y$$

$$0.15 \times 30 = 4.5$$

15% of 30 is equal to **4.5**.

When finding the per cent of a number, it is sometimes easier to multiply by a fraction than by a decimal.

$$\text{For example: } 10\% \text{ of } 50 = \frac{1}{10} \text{ of } 50 = 5$$

Learn About

Using Algebra: Per cents and Proportions

Some equations include **per cents**. Equations with per cents have three parts – the per cent, the base and the percentage. Equations with per cents can be written as **proportions** by placing the per cent over 100 and the percentage over the base.

Equation with a per cent	Proportion
$10\% \times 200 = 20$ <small>per cent base percentage</small>	$\frac{\text{per cent}}{100} = \frac{\text{percentage}}{\text{base}}$ <small>per cent percentage base</small>

To solve an equation with a per cent, follow these steps:

- Write a proportion using a variable as the unknown.
- Solve the proportion for the variable.

Look at the problem. Find the percentage.

Jill has 24 bracelets, and 25% of the bracelets are made of silver. How many of Jill's bracelets are made of silver?



First, write an equation with a per cent. Then change the equation to a proportion. Finally, solve the proportion for the variable.

$$\begin{array}{l} 25\% \times 24 = s \\ \frac{25}{100} = \frac{s}{24} \\ 25 \times 24 = 100s \\ 600 = 100s \\ 6 = s \end{array}$$

Jill has **6** bracelets that are made of silver.



Equations with **per cents** have three parts – the per cent, the base and the percentage. To solve an equation with a per cent, change the equation to a **proportion**.

Here are some common per cent and fraction equivalencies.

$10\% = \frac{1}{10}$	$20\% = \frac{1}{5}$	$25\% = \frac{1}{4}$	$33\frac{1}{3}\% = \frac{1}{3}$	$40\% = \frac{2}{5}$
$50\% = \frac{1}{2}$	$60\% = \frac{3}{5}$	$66\frac{2}{3}\% = \frac{2}{3}$	$75\% = \frac{3}{4}$	$80\% = \frac{4}{5}$

You can use the proportion equation to determine what percentage of 80 is 30 by substituting the known values into the equation. Substitute 80 for x and 30 for y .

$$\begin{array}{l} \frac{p}{100} = \frac{30}{80} \\ 80p = 30 \times 100 \\ 80p = 3000 \\ p = 37.5\% \end{array}$$

RESEARCH SUMMARY

The following is a summary of the research upon which the *FOCUS on Mathematics* series is based.

Overview

The *FOCUS on Mathematics* series is a targeted maths-strategy practice program geared towards both on-level and off-level maths students. The research summary is based on a literature review of academic monographs, journals and reports by content-area researchers and education experts.

The summary covers the following topics in support of the series *FOCUS on Mathematics*:

- Introduction to the Series
- What Is the Need for *FOCUS on Mathematics*?
- How Is *FOCUS on Mathematics* Supported by Research?
- How Does Research Support the Assessments Found in *FOCUS on Mathematics*?
- Quick-Reference Chart: From Research to Application: Strategies and Features in *FOCUS on Mathematics*

Introduction to the Series

FOCUS on Mathematics is a series designed for on-level and struggling maths students who need repeated practice. *FOCUS on Mathematics* centres on brief instruction and concentrated practice with targeted maths concepts and strategies in the context of word problems.

The *FOCUS on Mathematics* series covers:

Building Number Sense	Using Geometry
Using Estimation	Determining Probability and Averages
Using Algebra	Interpreting Graphs and Charts

What Is the Need for *FOCUS on Mathematics*?

There is a current drive in mathematics education to meet 21st-century skills so that today's students will be competitive in tomorrow's workforce. Several expert panels and mathematical organisations have sounded the alarm bell for improving students' mathematical understanding (e.g. NCTM, 2006; NMAP, 2008), as recent tests also show that students' mathematical progress is slowing (e.g. NCES, 2007).

In answer to these concerns about students' lacklustre mathematical performance, maths experts and researchers have joined forces to combat the slowing of mathematics progress.

The release of several major reports has named algebra as a "gateway to higher mathematics", which then leads to greater successes in both the academic and working lives of students (NCTM, 2006; NMAP, 2008). In response to this joint effort, these experts have also laid a pathway for students to follow in order to develop the mathematical skills and knowledge to master algebra. The *FOCUS on Mathematics* series may be an effective tool to help students along this pathway of proficiency to algebra.

The *FOCUS on Mathematics* series provides students with explicit instruction of key mathematical concepts and strategies combined with targeted practice in the context of word problems.

ANSWER KEY

FOCUS on Using Algebra, Book H

Lesson 1 (page 6)

1. B 2. C 3. C 4. B

5. Solution: Connor would have paid a total of \$292.50 without the sale. He saved \$58.50.

Sample Explanation: *To find how much Connor would have paid without the sale, I represented the problem as a proportion.*

$$\frac{80}{100} = \frac{234}{y}$$

I then solved for y by cross-multiplying.

$$\frac{80}{100} = \frac{234}{y}$$

$$80y = 234 \times 100$$

$$80y = 23,400$$

$$y = 292.5$$

To find how much he saved, I subtracted the amount he paid from the amount he would have paid without the sale.

$$\$292.50 - \$234.00 = \$58.50$$

Lesson 2 (page 8)

1. C 2. B 3. D 4. C

5. Solution: There were 100 students who chose a sport other than football.

Sample Explanation: *I wrote and solved a proportion to find how many students chose football.*

$$\frac{4}{5} = \frac{x}{500}$$

$$5x = 2000$$

$$x = 400$$

Then I subtracted the number of students who chose football from the number of students who took the survey to find how many students chose a sport other than football.

$$500 - 400 = 100$$

Lesson 3 (page 10)

1. D 2. D 3. B 4. C

5. Solution: Will will need to add 30 toothpicks to construct the tenth row. He will need a total of 165 toothpicks to construct the triangle with 10 rows.

Sample Explanation: *I studied the chart to find the pattern. Each number in the top row is multiplied by 3 to determine the corresponding number in the bottom row. To find how many toothpicks will need to be added to make the tenth row, I multiplied the number of rows (10) by 3 to get 30.*

To find the total number of toothpicks used to make a 10-row triangle, I completed the table.

Figure Number	1	2	3	4	5	6	7	8	9	10
Toothpicks Added to Make Bottom Row	3	6	9	12	15	18	21	24	27	30

I started with the first row and added the numbers of toothpicks needed to make each of the next rows. These numbers appear in the table.

1 row: 3	6 rows: 45 + 18 = 63
2 rows: 3 + 6 = 9	7 rows: 63 + 21 = 84
3 rows: 9 + 9 = 18	8 rows: 84 + 24 = 108
4 rows: 18 + 12 = 30	9 rows: 108 + 27 = 135
5 rows: 30 + 15 = 45	10 rows: 135 + 30 = 165

Lesson 4 (page 12)

1. D 2. B 3. C 4. B

5. Solution: Andrew's painting will cost \$314.

Sample Explanation: *First, I wrote an equation to show how Andrew prices his paintings.*

$$\$26h + \$2p = c$$

Then I substituted 5 for the hours (h) and 92 for the perimeter (p) and solved the equation.

$$(\$26 \times 5) + (\$2 \times 92) = c$$

$$\$130 + \$184 = \$314$$