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Expressions and equations

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Lesson 1 EXPONENTS

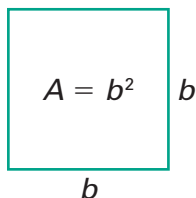
PART ONE: Learn about expressions with exponents



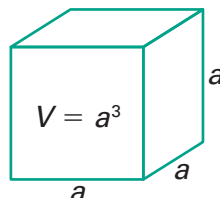
How can you simplify expressions with exponents?

Explore

The area of this square is $b \times b$, or b^2 .



The volume of this cube is $a \times a \times a$, or a^3 .



The 2 and 3 in the expressions above are called **exponents**.

Exponents can be used as a shortcut for representing repeated multiplication.

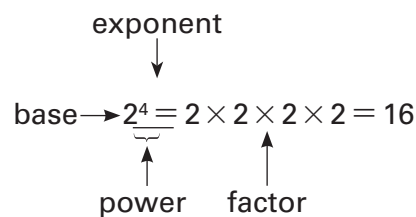
What are the rules for working with expressions involving exponents?

Think

An expression like 2^4 is called a **power**.

The **base**, 2, is a **factor** that is multiplied by itself.

The exponent, 4, is the number of times the base is used as a factor.



Use an exponent to rewrite $a \times a \times a$ as a power. a^3

Use exponents to rewrite $(a \times a \times a) \times (a \times a \times a)$. $(a^3) \times (a^3)$ or a^6

Use exponents to rewrite $\frac{a \times a \times a}{a \times a}$. $\frac{a^3}{a^2} = a$ or a^1

Use exponents to rewrite $(a^2)^3$. $(a^2) \times (a^2) \times (a^2)$ or a^6

Connect

The rules for working with exponents can help you simplify expressions with exponents:

- Add the exponents when *multiplying* powers with the same base. $a^3 \times a^3 = a^6$
- Subtract the exponents when *dividing* powers with the same base. $\frac{a^3}{a^2} = a^1$
- Multiply exponents when a power is *raised* to an exponent. $(a^2)^3 = a^6$

Let's Talk

What is any number divided by itself? Use the subtraction rule to find $\frac{a^2}{a^2}$.

Then use the result to make a general statement about n^0 for any number n .



Think It Through

Fill in the blanks as you solve the problem.

Simplify each expression.

$$b^2 \times b^5 \quad \frac{c^5}{c^2} \quad (d^2)^5$$

- The expression $b^2 \times b^5$ shows _____ two powers with the _____ base.

What should you do with the exponents? _____

Add the exponents. $2 + \underline{\quad} = \underline{\quad}$

Solution: $b^2 \times b^5 = \underline{\quad}$

- The expression $\frac{c^5}{c^2}$ shows _____ two powers with the _____ base.

What should you do with the exponents? _____

Subtract the exponents. $5 - \underline{\quad} = \underline{\quad}$

Solution: $\frac{c^5}{c^2} = \underline{\quad}$

- The expression $(d^2)^5$ shows _____ a power to an _____.

What should you do with the exponents? _____

Multiply the exponents. $2 \times \underline{\quad} = \underline{\quad}$

Solution: $(d^2)^5 = \underline{\quad}$

You can work with the expanded form of an expression containing exponents to check your answer.

$$\frac{c^5}{c^2} = \frac{c \times c \times c \times c \times c}{c \times c}$$

$$= c^3$$



Your Turn

Now, use what you know to solve this problem.

1. Simplify $\frac{m^2 \times m^6}{m^4}$. Show your work.

$$\frac{m^2 \times m^6}{m^4} = \underline{\quad}$$

PART TWO: Learn more about expressions with exponents



How can you factorise expressions with exponents?

Explore

You can use the **Distributive Property** and the rules for exponents to work with expressions with exponents.

The Distributive Property deals with expressions involving multiplication and addition or subtraction.

Distributive Property

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

The area of this rectangle is $n^2 + 5n$. What expression represents the length of the rectangle?

$$\begin{array}{|l} \text{Area} = \\ n^2 + 5n \\ \hline n \\ \hline ? \end{array}$$

Think

The area of a rectangle is equal to the length times the width.

Think:

$$\underline{\text{length}} \times \text{width} = \underline{\text{area}}$$

$$\underline{\text{length}} \times n = \underline{n^2 + 5n}$$

Connect

When each term of an expression has a **common factor**, you can use the Distributive Property to **factorise** the expression.

The terms of the expression on the right are n^2 and $5n$.

Factorise each term. Use the rules for exponents for n^2 : $n^2 = n^{1+1} = n^1 \times n^1$

Use the Distributive Property to factorise the common factor, n .

$$A = n^2 + 5n$$

$$A = \boxed{n} \times n + \boxed{n} \times 5$$

$$A = n(n + 5)$$

The factor n is common to both terms.

So, $A = n(n + 5)$, and also $A = \text{length} \times \text{width}$.

If the width of the rectangle is n , then the length must be $(n + 5)$.

Let's Talk

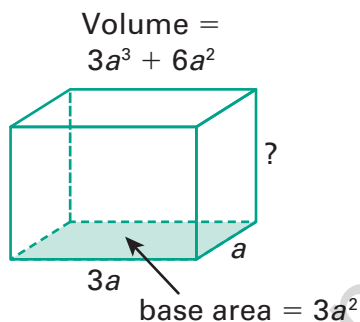
How can you use the rules for exponents and the Distributive Property to **expand** $x(x^2 + x + 1)$?



Think It Through

Fill in the blanks as you solve the problem.

The volume of this prism is $3a^3 + 6a^2$, and the area of its base is $3a^2$. What expression represents the height of the prism?



- The volume of a prism is equal to the area of its base times its height. Think:

$$\text{base area} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

- The terms of the expression $3a^3 + 6a^2$ are $3a^3$ and $\underline{\hspace{1cm}}$.

$$3a^3 = 3 \times a \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \quad 6a^2 = 2 \times 3 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$$

Circle all the matching factors to find the common factor of the terms.

$$3a^3 = 3 \times a \times a \times a \quad 6a^2 = 2 \times 3 \times a \times a$$

The common factor is $3 \times a \times a$, or $\underline{\hspace{1cm}}$.

- Use the Distributive Property to factorise the expression for volume.

$$3a^3 + 6a^2 = 3a^2 \times \underline{\hspace{1cm}} + 3a^2 \times \underline{\hspace{1cm}}$$

$$= 3a^2 (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$$

\uparrow \uparrow
 base area height

Solution: The expression $\underline{\hspace{1cm}}$ represents the height of the prism.

Factorising and expanding are opposite processes. Both use the Distributive Property.

Expand $n(n + 8)$:
 $n(n + 8) = n^2 + 8n$

Factorise $n^2 + 8n$:
 $n^2 + 8n = n(n + 8)$



Your Turn

Now, use what you know to solve this problem.

- Use the Distributive Property to expand $2b(b^2 + 4)$.

$$2b(b^2 + 4) = \underline{\hspace{2cm}}$$

PART THREE: Choose the right answer

Solve the problem. Then read why each answer choice is correct or not correct.

Solve

Which expression is equivalent to $a^2 \times a^4 \times a$?

- Ⓐ a^6
- Ⓑ a^7
- Ⓒ a^8
- Ⓓ a^9

Check

Check whether you chose the correct answer.

$$a = a^1, \text{ so } a^2 \times a^4 \times a = a^2 \times a^4 \times a^1.$$

Each of the powers has the same base.

To multiply powers with the same base, add the exponents.

$$a^2 \times a^4 \times a^1 = a^{2+4+1} = a^7$$

$$2 + 4 + 1 = 7$$

So, the correct answer is Ⓑ.

Why are the other answer choices not correct?

Ⓐ a^6	The exponent of the last factor, a , was neglected. Because $a = a^1$, the exponents are 2, 4 and 1.
Ⓒ a^8	The exponents should be added, not multiplied.
Ⓓ a^9	The first and second exponents were multiplied and then the third exponent was added. All three exponents should be added.


Your Turn

Solve each problem. Use the hints to avoid mistakes.



- If a variable has no written exponent, its exponent is 1. For example, $a = a^1$.
- To *multiply* powers with the same base, *add* exponents; do not multiply exponents.
- To *divide* powers with the same base, *subtract* exponents; do not divide exponents.

3. Which is a common factor of all three terms in the expression below?

$$4b^3 + 2b^2 + 6b^4$$

- (A) b^3
- (B) b^4
- (C) $2b^2$
- (D) $2b^3$

4. Which operation should you perform on the exponents to simplify the expression?

$$\frac{x^6}{x^2}$$

- (A) addition
- (B) division
- (C) multiplication
- (D) subtraction

5. A rectangle has an area of $m^3 + 2m^2$ and a width of $m + 2$.



$$A = m^3 + 2m^2 \quad m + 2$$

What is the length of the rectangle?

- (A) m^2
- (B) m^3
- (C) $(m^3 + 2m^2) - (m + 2)$
- (D) $(m^3 + 2m^2) + (m + 2)$

6. Which shows a pair of expressions that are equivalent?

- (A) $(p^2)^3$ and p^5
- (B) $\frac{t^8}{t^4}$ and t^2
- (C) $q^4 + q^2$ and q^6
- (D) $y^2 \times y^2 \times y$ and y^5

PART FOUR: Write the best answer

Study the model. It is a good example of a written answer.

Student model

Show

The base of a rectangular prism has an area of $2x^2$.

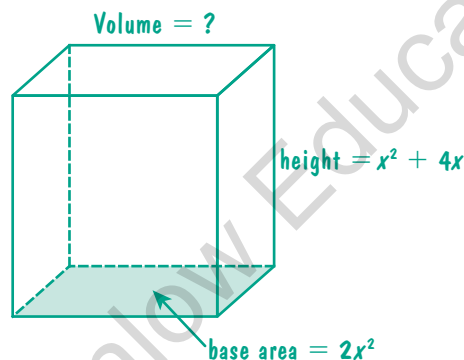
The height of the prism is $x^2 + 4x$.

Write an expression, without brackets, that gives the volume of the prism.

Show each step. Then explain how you found the solution.

Volume = base area \times height

$$\begin{aligned} V &= 2x^2(x^2 + 4x) \\ &= 2x^2(x^2) + 2x^2(4x) \\ &= 2x^2(x^2) + 2x^2(4x^1) \\ &= 2x^{2+2} + (2 \times 4)x^{2+1} \\ &= 2x^4 + 8x^3 \end{aligned}$$



The student shows each step.

The student correctly answers the question asked.

The student gives important details about how to find the answer.

The student uses the maths words *Distributive Property, expand, powers, base and exponents.*

Solution: The volume of the prism is $2x^4 + 8x^3$ cubic units.

Explanation:

I drew a rectangular prism and labelled it with the given height
and base area to help me see the problem. I knew that the volume
of a prism is the base area times the height, so I wrote an expression
for the volume, using the given expressions for the base area and
the height. Then I used the Distributive Property to expand the
volume expression, using the rules for multiplying powers with the
same base (add the exponents).

Explain

Your Turn

Solve the problem. Use what you learned from the model.

7. A rectangular playing field has an area of $2z^3 + 6z^2$.

The width of the field is $z + 3$.

What is the length of the field?

Show each step. Then explain how you found the solution.

**CHECKLIST**

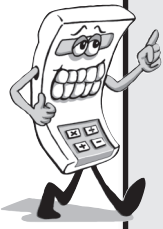
Did you . . .

- show each step?
- answer the question asked?
- give important details?
- use maths words?

Solution: _____

Explanation:

PART FIVE: Prepare for a test



As you solve problems involving exponents, remember to:

- add exponents to multiply powers with the same base.
- subtract exponents to divide powers with the same base.
- multiply exponents to raise a power to an exponent.
- use the Distributive Property to factorise or expand an expression.

Solve each problem.

8. Which of the following expressions is equivalent to the expression below?

$$x(x^2 + 4)$$

- (A) $x^2 + 4x$
(B) $x^3 + 4x$
(C) $x^3 + 4$
(D) $x^2 + x + 4$
9. A prism has a volume of $b^3 + 2b^2 + b$. The height of the prism is b . What is the area of the base?

- (A) $b^2 + 2b + 1$
(B) $b^2 + 2b + b$
(C) $b^3 + 2b^2 + b$
(D) $b(b^2 + 2b + 1)$

10. Which expression can be simplified by multiplying the exponents?

- (A) $\frac{t^6}{t^3}$
(B) $(c^4)^3$
(C) $q^2 \times q^5$
(D) $r^3 + r^5$

11. Which expression has terms with a common factor of a^2 ?

- (A) $2a + 2a^2$
(B) $4a^3 + 5a$
(C) $5a^3 + 3a^2$
(D) $6a^3 + 2a^2 + 3$

12. A rectangle has length $5z$ and width $z^2 + 3z$. What is the area of the rectangle?

- Ⓐ $z^2 + 8z$
- Ⓑ $z^3 + 3z^2$
- Ⓒ $5z^3 + 3z$
- Ⓓ $5z^3 + 15z^2$

13. Which expression is equivalent to $y^8 \times y^4$?

- Ⓐ y^2
- Ⓑ y^4
- Ⓒ y^{12}
- Ⓓ y^{32}

14. Look at the expression below.

$$\frac{n^{12}}{n^3}$$

What operation should you perform on the exponents to divide the powers?

Divide the powers.

15. A rectangular prism has a volume of $3a^4 + a^2$. The area of the base of the prism is $3a^2 + 1$. What is the height of the prism?

Show each step. Then explain how you found the solution.

Solution: _____

Explanation:
