MAKING **MATH** STICK

Classroom Strategies that Support the Long-Term Understanding of Math Concepts

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Introduction

Teachers across classes, grade levels, and schools have been seeing the same problem. What we have here is a serious educational issue.

We have all experienced it: students learning math concepts only to forget them in the weeks and months that follow. The first time it happened may have seemed strange, but it now seems like a real problem. Students are not retaining what they learn. If we noticed just one or two students unable to recall previous learning, that might be an individual student issue. But that is not the case. Teachers across classes, grade levels, and schools have all been seeing the same problem. What we have here is a serious educational issue.

We know that teachers put their hearts into their work. We also know that school districts are revising math curricula and offering multiple professional learning opportunities to address the problem of students not retaining what they have learned. Unfortunately, we continue to see students unable to apply previous learning to novel situations. Our way to approach this has always been to stop and re-teach math concepts. In some cases, this re-teaching resembles cramming, which doesn't work in the long run and uses up valuable class time.

What is causing this lack of recall? It can't be the topic. Past generations of students have successfully learned mathematics. It can't be the time put in. Teachers are probably working harder than in times past. It could be the digital age we live in, which discourages people from putting information to memory. Few of us attempt to remember things because everything we need to know can be accessed easily via the Internet. Does this new inability to retain learnings have something to do with how we support our students?

Whatever the cause, I believe that we can address the issue effectively. I know because I have turned things around in my own classroom. The solution is not to work harder but to work smarter. We, as educators, can shift our paradigm from "teach, test, move on" to "teach, connect, apply," a move that can optimize student learning opportunities. This book is intended to provide you with an understanding of simple, manageable, and sustainable strategies you can use to work smarter.

Whether we are classroom teachers, coaches, administrators, coordinators, or consultants, we all have a lot on our plates. All too often we have tasks added to our plates while none are removed. I do not wish to add to your workload.

Instead, I hope to offer you a method for meeting the varying needs of your learners in a manageable and sustainable yet effective way. In the end, you will not have more work. You will simply be working differently.

The instruction and learning strategies I offer were developed based on evidence, in terms of both theory and classroom practice. They are straightforward—you could apply many of them in your classroom tomorrow. So that you can see these strategies at work in real classrooms, I have provided accounts from teachers and students sharing their experiences. You will also see work samples showing the strategies at work.

This book is meant to be one that can stay on your desk so that you can refer to it throughout the year. The strategies it offers are applicable to all mathematical concepts and classrooms. It is meant to support you in reaching whatever goals you establish for your particular students.

Why We Need a Different Approach to Learning

So now to the classroom.

The following snapshots are taken from conversations with educators about their experiences in the classroom. As you read them, consider if you have experienced anything similar in your classroom.

These conversations are composite conversations typical of many I have had with teachers at all three levels. This book is filled with multiple examples of student work, as well as excerpts from conversations with teachers and students. Although they were contributed anonymously, all excerpts come from real conversations with real people.

PRIMARY EXAMPLE: A TEACHER EXPLAINS

Forgetting how to estimate using a referent

I spent a considerable amount of time in the fall introducing estimation. More specifically, it was estimating with a referent. At first, we used concrete objects but then, over time, moved to pictures. It is now the winter and, when asked to estimate, my students will count the items and then record that as their estimate. I have tried, on many occasions, to encourage my students to use referents and have even shown a picture of a referent beside the items to be estimated. Still, no luck. I am at a loss and cannot spend any more time on this. Why were they able to demonstrate success with estimation using referents in the fall, but now it seems like they never experienced it? They had it so well but now say that they don't remember doing it.

ELEMENTARY EXAMPLE: A TEACHER EXPLAINS

Forgetting how to work with prime and composite numbers

I was so impressed with my class for their work on prime and composite numbers. Students were able to identify which numbers were prime and which were composite when provided a list. Taking it further, students could identify the common factors for two given composite numbers and could list common multiples for given prime numbers. I would structure the lessons as follows: introduction, minilesson of a specific concept, and then practice. Students would be provided plenty of practice questions to apply their understanding of the concept in the minilesson. From there, we had a unit test, and all students performed very well. It has now been a few weeks, and I asked students to solve a few problems on prime and composite numbers. I was shocked! They were mixing up prime and composite numbers and were getting factors and multiples confused. What is going on?

INTERMEDIATE EXAMPLE: A TEACHER EXPLAINS

Forgetting how to solve equations

Solving equations is an important part of the Grade 8 math curriculum. So we spent a lot of time on this. At the end of the unit, students could solve equations involving a single variable. I assessed students and determined that they could isolate a variable whether it was added, subtracted, multiplied, or divided. Now, after March break, students cannot isolate the variable if it is multiplied or divided with another number. In fact, much of the class is saying this is hard and that they need step-by-step support when working on such equations. Where did all their success earlier in the year go? I thought that they understood what to do.

As you were reading the snapshots from the various classrooms, did you think back to times that you experienced similar unwelcome surprises? It can be quite perplexing and frustrating to realize that, although you spend a considerable amount of time on a concept, instructing and assessing, students seem to lose this learning as time goes on. You may have even communicated through report cards that students have demonstrated grade level understanding only to find out you need to review or re-teach a concept.

Looking back, what could you have done differently? You may be thinking, "I introduced the concept, provided students with multiple tasks to address and practice the concept, formatively assessed student thinking, addressed misconceptions, and then re-assessed to evaluate understanding. It seems that I have done all that I could do to address student learning. So why are all my efforts not working?"

Perhaps we should pause and think about why student failure to recall is becoming the norm. Are we okay with this being the norm?

The issue of poor retention is creating problems as students move from grade to grade. You have probably heard fellow teachers complain about having to re-teach concepts supposedly taught in the previous grade because students say that they either forget the concept or don't remember working on it the year before. While we sometimes accept this as normal, perhaps we should pause and think about why student failure to recall is becoming the norm. Are we okay with this being the norm? Should we be trying to address it? And how could we do so?

Considering the One-and-Done Approach to Teaching Math

I would like to suggest, at this point, that the way that we structure the teaching of math may hold great potential for improving student recall. Consider the standard one-and-done organization of most math textbooks and year plans. Concepts are isolated into units and then each unit is segmented into smaller lessons. This is a great plan if you want to support short-term performance.

The following is a visual of a one-and-done year plan:

Topic A → Topic B → Topic C →

Blocked practice

Of significance is that the year plan is straightforward and linear. There is no returning to previously addressed mathematical concepts. Instead, the teacher covers Topic A, moves on to Topic B, and then to Topic C. Such an approach to instruction and learning is practice based on units of study. Such practice is referred to as blocked practice (Rohrer, 2009).

Consider how students would typically learn geometry in a Grade 8 classroom. The geometry unit has been compartmentalized into segments such as sketching a view of an object, drawing rotated objects, constructing objects from a certain perspective, identifying transformations, constructing tessellations, and identifying transformations in tessellations. During the first class, students receive a minilesson or exploratory opportunity on one of the segmented concepts. In the next class, they learn about a different segmented concept, in isolation. The pattern continues. While each of the segments plays a crucial part in student spatial reasoning, they are all treated individually. Students receive instruction during each lesson and are then assigned practice questions on the specific aspect of geometry addressed in the lesson.

In blocked practice, students are not required to consider which strategy to apply as this is already inherent in the questions and tasks assigned—all questions and tasks apply only to the single concept that was the focus of the lesson. A visual representation of this idea might be as follows.

Lesson Topic A
Practice questions:
1. Topic A 2. Topic A 3. Topic A 4. Topic A

Lesson Topic B
Practice questions:
1. Topic B 2. Topic B 3. Topic B 4. Topic B

Lesson Topic C
Practice questions:
1. Topic C 2. Topic C 3. Topic C 4. Topic C

Question order typical of blocked practice

The one-and-done structure of most mathematical teaching does a significant disservice to students.

Notice that all practice questions relate directly to the focus of the lesson just taught. This removes the requirement that students select a strategy to apply to any given question or task. This drawback of the one-and-done structure of most mathematical teaching does a significant disservice to students. Missing are the cognitive processes that students would use in determining an appropriate strategy. These processes are crucial to mathematical understanding and problem solving.

Consider the potential impact of learning individual mathematical concepts in isolation through the course of a school year. While math is a network of concepts in which there is much interconnectivity, the one-and-done school year is comprised of segmented learning. Instead of working through the intricate tapestry of mathematics, concepts are compartmentalized.

Student Learning Strategies		
Strategies	**In the Classroom**	**Examples Explored**
Chapter 1: Learning through Self-Assessment	Help your students to use objective tools to gain a realistic perspective of what they don't know yet and consolidate what they do know.	Students • create their own **self-quizzes** • use **flashcards** for mixed and spaced review • **self-monitor**
Chapter 2: Building a Network of Memory	Help your students elaborate their knowledge of a concept by adding layers of meaning to a memory.	Students • create and answer **how and why** questions • **explain** their thinking • use **dual coding** • **compare and contrast** concepts • generate **concrete examples** of abstract ideas • **make connections** to prior knowledge
Chapter 3: Learning by Figuring It Out	Help your students approach problems without first having a defined "correct" strategy.	Students • use their **mistakes** to learn • **surf** problem-solving strategies • **predict and check** answers to a problem • **work backward** from a given solution
Chapter 4: Learning by Picturing It	Help your students create images to explore problems and strengthen memory.	Students • **visualize** a problem to "see" it • **free sketch** to explore a problem • create **concept maps** to find connections and relationships • create **graphic organizers** to organize thinking
Chapter 5: Learning by Writing	Help your students use informal, exploratory writing to help them think through mathematical concepts.	Students • **paraphrase** a problem to help them understand it • approach a problem by doing a **freewrite** • do a freewrite of **30 words or less** • use a **plus/minus chart** to record what they know and don't know
Chapter 6: Using Awareness Strategies to Improve Learning	Help your students consciously improve their learning process.	Students • **prioritize** aspects of a problem • **summarize** a problem-solving experience • **reflect** on a problem-solving experience • use a **learning journal** to record key takeaways • record their **next steps**

CHAPTER 1

Learning through Self-Assessment

Help your students use objective tools to gain a realistic perspective of what they don't know yet and to consolidate what they do know.

When we are on a journey, it is important to have a clear idea of our starting point, our destination, and our means of transport. There is no point in rushing out the door unless we know in which direction to travel. And, as we travel, it helps to know where we are, so that we can stay on track. What we need is a global positioning system (GPS).

Too many students appear to be lacking a GPS for their learning journey.

Too many students appear to be lacking a GPS for their learning journey. They may identify, or be told, what the learning goal is for a particular lesson or unit but be unsure where they are in relation to that goal. Students can better plan their route to mastery of a concept if they know what they know about the concept and what they do not know yet. This understanding must be objective and not based on inaccuracies. For students, the GPS they need for learning is effective self-assessment. Through ongoing self-assessment and feedback, students can gain a clear and accurate picture of their status and what their next steps should be.

Self-assessment (also referred to as calibration) is the act of using an objective tool to gain a realistic perspective of what we know and what we do not know (Brown, Roediger III, & McDaniel, 2014). By aligning our self-assessment with *objective* feedback, we avoid illusion of mastery.

To make self-assessment useful, we must guide students in using objective assessment tools. Using an objective tool enables students to avoid illusion of mastery because—like all of us—students are susceptible to inaccurate notions of what they know. When you provide students with objective tools, you will be helping them to get an accurate picture of where they stand.

To be effective, self-assessment tools must also provide timely feedback. Too often, when studying or practicing a strategy, students will move forward through countless problem sets without identifying if or where they went wrong. What this does is create two fallacies. First, without knowing if their responses are correct or incorrect, students will not have an accurate gauge of their understanding. They may think that they are strengthening their understanding of a concept when in fact they are not. What they are doing, instead, is reaffirming an inaccurate application of a strategy. Second, without checking their responses for accuracy, the student is forming a path not toward their learning goal but away from it.

Thus, to strengthen their ability to apply previous learning, students need assessment tools that provide feedback that is both objective and timely. Objective self-assessment tools can include self-quizzing, flashcards, study guides, question sets, self-monitoring, and so on.

Timing is key to successfully using self-assessment tools for learning. Taking a self-assessment directly following an initial lesson is not terribly helpful. The point should not be to measure short-term performance. Self-assessing is most useful when we take stock of our status after some time has passed, to assess what learning has "stuck."

Retrieval practice that is spaced and mixed provides students with the best opportunity to accurately self-assess. When time has passed and concepts are mixed, the learner is forced to think about the prompt, retrieve previous learning, reconsolidate knowledge, and then apply it to answer the question. This form of self-assessment will inform students of their progress in relation to their learning goals. It's about taking stock of their *retention* of understanding.

Self-assessment tools are useful not only for *measuring* learning but for making that learning happen.

Self-assessment tools are useful not only for *measuring* learning but for making that learning happen. While using a self-assessment tool may take more effort than rereading texts or notes, the greater effort exerted by the student will strengthen both the pathways to previous learning as well as the previous learning itself.

By self-assessing, students give themselves immediate feedback that can guide them in next steps. This may involve focusing on certain topics or changing the type of questions to practice. Self-assessment is an accurate guide for students to support them in becoming independent learners.

Some students may be able to create and answer questions that focus on the main ideas of a concept. Others will struggle to create suitable questions, so you may want to supply questions to prompt their thinking. After completing the self-assessment and identifying weaknesses, students can then address the gaps in learning with additional work.

Self-assessment tasks such as self-quizzing are powerful because students must work through desirable difficulties to recall previous learning and apply it to new contexts. To make this work, these tasks must be spaced over a period of time and must mix up the tasks being reviewed so that the same strategy isn't the focus of a single self-assessment experience.