MESMERIZING MATH PUZZLES!

Lydian Davis

Hawker Brownlow EDUCATION

TABLE OF CONTENTS

CHAPTER 1: TEACHER/PARENT GUIDE

PART 1: COMMON CORE STATE STANDARDS

The puzzles in this book are designed to be easily integrated into any curriculum, especially one based on the Common Core State Standards for mathematics. As recommended by the Common Core State Standards Initiative, the puzzles were created to facilitate "[connecting] the mathematical practices to mathematical content in mathematics instruction." Each of the two areas (content and practices) will be discussed separately below, but they cannot be separated in the process of solving the puzzles.

The following chart summarizes the specific content standards covered by each type of puzzle. Puzzle types are listed in order of increasing difficulty. Standards are indicated for the recommended grade level and up.

	BTN	LS	PP	CN	KEN	AL	KAK	BZ	TF	TS
4.OA.A	XX	XX	XX							
4.NBT.A	XX	XX								
4.NBT.B	XX	XX								
5.OA.A				XX		XX				
5.NBT.A	XX	XX								
5.NBT.B	XX	XX	XX	XX	XX	XX	XX	XX	XX	XX
6.NS.B	XX	XX	XX	XX	XX	XX	XX	XX	XX	XX
6.EE.A.2	XX					XX			XX	
6.EE.B.5	XX	XX			XX	XX	XX	XX	XX	XX
6.EE.B.6		XX	XX	XX	XX		XX			XX
6.EE.B.7		XX	XX	XX	XX		XX	XX		XX
6.EE.B.8	XX	XX	XX	XX	XX	XX	XX	XX	XX	XX

Puzzle names are abbreviated as:

BTN = By the Numbers
LS = Lost Sums
PP = Picture Puzzles
CN = Cross Numbers
KEN = KenKen
AL = Addition Logic
KAK = Kakuro
BZ = Buzzippers
TF = Transformers
TS = TriSquares

In my earliest experience with gifted elementary students, I learned that they master content more quickly and more effectively when it is embedded within exercises requiring reasoning. Puzzles provide the perfect format in which this mastery can take place. Each of the puzzles in this book provides practice in the content standards. And every puzzle on every page requires the use of at least one of the practice standards in order to solve it. Following is a brief discussion of how the puzzles relate to each of the Standards for Mathematical Practice.

MP1. "Make sense of problems and persevere in solving them. . . analyze. . . plan a solution pathway. . . monitor and evaluate their progress. . . identify correspondences between different approaches."

By their very nature, gifted children are drawn to puzzles. They enjoy the challenge of making sense of the problems and seeking a solution that requires more than simply performing calculations. Thus, they will be motivated to persevere in solving these puzzles. In addition, the "Helps" provide just enough support to enable them to persevere when the going gets tough.

Analysis and planning are essential to the solving process. The puzzle format allows the students to monitor and evaluate their own progress – if one answer is incorrect, it will impact another answer within the same puzzle. Every puzzle allows for different approaches, providing a platform for the solvers to discuss those approaches and analyze where they correspond.

MP2. "Reason abstractly and quantitatively. . . make sense of quantities and their relationships . . . attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations."

Each puzzle in this book requires abstract and quantitative reasoning in addition to computation. But the reasoning used is not simply a means of solving an isolated puzzle; it is designed to deepen the child's understanding of quantities and their relationships, providing a foundation that the students will continue to build on as they progress to higher and higher levels of mathematics.

While completing the puzzles, the solvers will need to exercise flexibility in using different properties of operations. They will gain a greater appreciation for the meaning of those properties as they apply them to solving the problems.

MP3. "Construct viable arguments and critique the reasoning of others. . . make conjectures. . . justify their conclusions, communicate them to others. . . listen [to] or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments."

These puzzles are a perfect vehicle for strengthening elementary gifted children's abilities related to this standard. Every puzzle requires building viable arguments and critiquing mathematical reasoning. Because each of them can be approached from a variety of angles, there will be rich opportunities for groups of any size to discuss their arguments and conclusions, determine if they make sense, and ask a variety of questions related to those arguments.

MP4. "Model with mathematics. . . apply the mathematics they know to solve problems arising in everyday life, society, and the workplace."

None of the problems presented in these puzzles is based on a problem in everyday life. However, by solving the puzzles, the students will build mathematical reasoning strategies that will serve them for the rest of their lives, enabling them to apply those strategies to problems in life, society, and the workplace long after an individual puzzle has been successfully solved.

MP5. "Use appropriate tools strategically."

While solving the puzzles, the children should be encouraged to use the tools available to them, such as a calculator. A chart of the basic multiplication facts will be a helpful tool in finding the patterns that lead to the answers in some of the puzzles.

MP6. "Attend to precision. . . communicate precisely to others. . . calculate accurately and efficiently."

These puzzles provide many opportunities for the children to explain their reasoning to each other. In the process, they will learn to use greater precision and clear definitions in order to make themselves understood.

The pleasure of solving the puzzles will motivate the students to become more accurate and efficient in their calculations. In my first experience in teaching mathematically gifted third graders, some of them were resistant to learning the basic multiplication facts because it seemed so tedious. They continued to use repeated addition to determine the products that they had not yet memorized, long after they should have known the facts by heart. Being gifted, they were quite capable of learning them much more quickly, but (being gifted!) they resisted making the effort.

When we moved on to long division, they were suddenly motivated to commit the basic facts to memory in record time. In the same way, since puzzle solving is a challenging and enjoyable activity for this population and since one error can impact many different parts of the same puzzle, the students will be motivated to calculate more accurately and efficiently in order to solve the puzzles correctly.

MP7. "Look for and make use of structure. . . discern a pattern or structure."

Discovering the patterns in the mathematics needed to solve the puzzles is essential to finding the correct solutions. Two examples are: finding the patterns in calculating with odd and/or even numbers, and observing that the greatest amount that can be carried when adding a column of numbers is one less than the number of addends in the column.

MP8. "Look for and express regularity in repeated reasoning. . . look both for general methods and for shortcuts. . . maintain oversight of the process, while attending to the details."

These puzzles provide many opportunities for children to discover regularity in repeated reasoning. For example, in solving "Picture Puzzles" the students will use repeated reasoning to develop shortcuts for determining which squares must be filled in and which ones must be empty.

Solving each of the puzzles requires maintaining oversight of the process while attending to the details. This will occur both within the individual problems that make up a puzzle and within the puzzle as a whole.

All citations are from the Common Core State Standards Initiative website. Copyright 2010, National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved. Accessed January 2014.

PART 2: USING THE PUZZLES

Warning – These puzzles are tough! They will challenge even the most mathematically gifted sixth graders. But then, that kind of challenge is just what these students want and need.

GRADE LEVELS

General grade level recommendations are given below. But as you know, gifted children vary widely in their degree of giftedness and experience. Some sixth graders with little experience in mathematical reasoning will be best served by starting at the beginning of the book and working their way forward, possibly skipping a few puzzles here and there as they sharpen their reasoning skills. Some highly gifted fourth graders will need to begin with the puzzles recommended for fifth or sixth graders in order to find a suitable challenge.

I've suggested using the puzzles with fourth through sixth graders because most mathematically gifted children in those grades will find a large number of puzzles at an appropriate level, making this book a worthwhile investment for you as their teacher or parent. Mathematically gifted third graders are likely to be able to solve a handful of the puzzles, but their options will be more limited.

General grade level recommendations:
Grades 4 and up: By the Numbers, Lost Sums, Picture Puzzles
Grades 5 and up: Cross Numbers, KenKen, Addition Logic, Kakuro
Grades 6 and up: Buzzippers, Transformers, TriSquares

GROUPING

The puzzles can be used as an individual activity, in small groups, or with an entire class. When used as an individual activity, they will be helpful in situations such as homeschooling or the regular classroom, in which there are a limited number of gifted students who need an extra challenge.

When an individual within the regular classroom is pursuing a more difficult activity than his peers, a problem can occur if he gets stuck and is unable to progress on his own. He needs help, but it can be difficult for the teacher to provide assistance, when the other children are working on a different activity at the same time. The "Helps" have been designed to reduce the stress on both student and teacher by granting the student greater independence. He can be given the appropriate set of "Helps" as needed, to provide just enough direction to enable him to solve the puzzle successfully.

Small groups of students can work together to complete individual puzzles. Each group can work on a puzzle at its own level of difficulty and interest. The children can help each other to find the best starting points and discuss various approaches to solving the puzzles among themselves. If they reach an impasse, they can use the "Helps" to trigger ideas that lead to the solutions.

Discussing the puzzles in large group settings will contribute to growth in many of the Common Core Standards for Mathematical Practice. The students can consider various approaches to solving a puzzle, justify their reasoning, and critique the reasoning of others. If needed, the "Helps" can be used to initiate discussions.

PART 3: INTRODUCTION TO EACH PUZZLE TYPE

One of the challenges that puzzles present to elementary students is their messiness. They often resist the neat-niks' attempts to proceed from left to right and top to bottom, filling in each answer as they go along. Some gifted children are especially distressed by the need to move on before getting the complete answer, while others delight in the freedom of skipping around. You might need an extra dose of patience with the first group, but once they experience the pleasure of completing a puzzle, they will be more willing to work outside their box.

BY THE NUMBERS

I adapted these puzzles from others that I'd seen in teaching materials. In those sources, they were primarily missing addend exercises that did not require any higher level thinking skills. I've intentionally made them more challenging. I also added the rule that if a number appears in the puzzle, none of the letters in the puzzle will represent that number. These were one of the first types of puzzles that I made for Dell Puzzle Publications' magazines.

These puzzles are especially appropriate for developing flexibility in using the inverse operations of addition and subtraction and for discussing fact families. Students should be encouraged to think of the top number in the subtraction puzzles as the sum of the other two numbers. Those two numbers would then be thought of as addends.

LOST SUMS

As far as I know, Dell is the only publisher of these puzzles. I have been constructing them for Dell for about 20 years. The puzzles here follow the same format as those that I have sold to Dell, but include additional clues to make them appropriate for elementary solvers.

These puzzles can be used to extend students' understanding of place value. In most of the puzzles, it will be necessary to analyze the need for regrouping in each place. For example, if the ones digit in the sum is a 4, will it represent a total of 4, 14, or 24? The lowest possible sum of three different counting numbers is 6, so it can't be 4. The only way it can be 24 is if the three addends are 7, 8, and 9. If any of these three digits appears elsewhere in the puzzle, or if one of the digits is given and it is lower than 7, then the only possible sum will be 14. This regrouping will need to be taken into account when determining the missing addends in the tens column.

As an extension activity, ask the children to find the sum of the three numbers given on the right-hand side of the puzzle. It will always be 45 because it is the sum of the digits 1 through 9. Then have them find the sum of the digits in the puzzle's sum. It will always be a multiple of 9 because it is the same as adding the digits 1 through 9 with some regrouping. Any series of nine consecutive numbers will add up to a multiple of 9. Will this be true of any arithmetical series of nine numbers, like nine consecutive odd numbers? What about a series of five numbers, or three, or six?

PICTURE PUZZLES

These popular puzzles can be found in GAMES magazine and on the internet under various names, including "Paint by Number" and "Nonogrids." While I have constructed them and used them in gifted summer classes, I have not attempted to sell mine to either of those sources. The online puzzles are often computer generated and therefore do not produce a picture when completed.

Some of the gifted children that I've worked with have been resistant to spoiling the appearance of the final picture by inserting dots or circles in the spaces that they have determined must be empty. But those dots or circles are essential to solving the puzzles. For students whose perfectionistic tendencies may prevent them from finding the solution, I would recommend giving them two copies of each puzzle. They can mark up one copy as needed and enter only the filled spaces in the second copy, thus preserving a more pristine appearance.

While the basic math required may seem too simple for gifted children, the reasoning can be rather difficult. Counting the squares individually can be done to determine which ones to fill in, but the students should be encouraged to look for patterns and to use addition and subtraction instead.

As an extension activity, ask the children to construct ratios, proportions, or fractions to compare the number of spaces to be filled in to the total number of spaces remaining in a given row or column. How large does that fraction need to be in order to determine which space or spaces must be filled in? Can they formulate an appropriate rule?

Cross Numbers

Cross Numbers appear in a variety of puzzle magazines and math puzzle websites. In some sources, one or more digits can be used more than once in a puzzle, while others do not appear at all. The list of numbers is then given below each puzzle. I've chosen to use each of the digits from 1 to 9 exactly once in each puzzle. This makes the puzzles a little easier for elementary solvers.

Cross Numbers do not follow the "order of operations" rules, but they could be used to begin a discussion of those rules. After a puzzle has been solved, ask the children how the answers would vary if they used those rules.

KenKen

These puzzles can be found online and in puzzle books and magazines. They are a variation on the popular Sudoku, in that every row and every column contains each digit exactly once. But Sudoku is a logic puzzle, with no math required, while KenKen includes the use of mathematical reasoning. Most KenKen puzzles are 6 x 6, especially those intended for children, but they can be found in other sizes as well.

As an extension activity, students can explore the relationship between odd and even addends and factors. If the sum of two numbers is odd, what does that tell you about the addends? What about the sum of three numbers? What do you know about the product if the quotient is even?

Addition Logic

This is one of my favorite puzzles in the puzzle magazines that I buy. Those resources provide only five words in the list. I've increased the number of words to nine to adapt them to an elementary level. I've only seen them in Dell puzzle magazines, but they could be available from other sources as well.

Students might enjoy creating their own Addition Logic puzzles. It's not as easy as making a list of words that share the same nine letters. They will have to consider factors such as which letters should have the highest or lowest values in order to make the puzzles solvable. They can then try solving each other's puzzles, or make their own puzzle magazine to share with friends and family members.

Kakuro

Contrary to popular belief, this puzzle was not invented in Japan. It was invented by an American and first published in Dell puzzle magazines in the 1950's under the name "Cross Sums." For many years, Dell's only supplier was a woman. But Japan is known for its love of math and logic puzzles, such as Sudoku (which was also invented in the United States and called "Number Place"), while Americans have always preferred crosswords and other word puzzles. Why is that?

It's impossible to make a crossword puzzle using Japanese characters. In a crossword, the words must be broken down into small units such as individual letters in order to form a grid of intersecting words. Japanese characters each contain a great deal more information than a single letter of the alphabet. Their words cannot be broken down into enough small units to make a crossword puzzle. Other types of word puzzles such as Anacrostics present similar problems. So the Japanese have developed an interest in math and logic puzzles instead.

Kakuro is probably the most popular math puzzle in the world. They can be found in many books, magazines, and websites. Several variations are also available. Some publishers of Kakuro puzzles include a list of the combinations of addends for each possible sum. But that turns it into a logic puzzle. The mathematical reasoning comes in when the children have to figure out for themselves, for example, how many combinations there are of three different one-digit numbers that add up to 21. As your students successfully solve more and more Kakuros, they will remember more and more of the combinations and it will become more of a logic puzzle. But in the beginning, they should be required to find those combinations on their own.

As an extension activity, start a discussion on variations on the Kakuro rules. What if 0's were allowed? Would that make the puzzles easier or harder? What if you could repeat a digit in an answer? What if the puzzles used multiplication instead of addition? In that case, would you want to allow repeated digits?

BUZZIPPERS

I invented these puzzles based on the number game "Buzz Zip," which we used to play in elementary school math classes when I was a child. I don't know if teachers use the game anymore, but you can teach it to your class when you introduce the puzzles.

In the game, the students count one by one, beginning with "1," and continuing around the classroom repeatedly until all the children except one have been eliminated. A target number is chosen before the game begins. For each player's turn, if the number to be named is a multiple of the target number, she says, "buzz" instead of the number. If the number contains the target number as a digit, she says, "zip." If both of these conditions are true, she says, "buzz zip." If she makes a mistake, she is eliminated. For example, if the target number is 3, the counting would go: "1, 2, buzz zip, 4, 5, buzz, 7, 8, buzz, 10, 11, buzz, zip, 14. . . ." (In this case, navigating the 30's will be interesting!)

The counting continues as high as the class can go until all but one student have given the wrong response and thus have been eliminated. In recent years, when I have played this game with gifted children, those who are "out" have enjoyed following the play to try to catch the next mistake, thus keeping the entire group actively involved in the game.

The most obvious extension activities for these puzzles would be related to divisibility rules. In my first years of teaching elementary gifted math, I created the following poem to help my students remember those rules:

2: I may be small, but I'm powerful, you'll see.
Every even number is divisible by me.

3: My rule is simple and simply divine.
Sum the digits of the number to get 3, 6, or 9.

4: Millions, billions, zillions or more – most of the numbers can be ignored.
Just look at the last two digits – are they divisible by 4?

5: If you're looking for something easy, I'm your hero.
When it's divisible by me, a number ends in 5 or 0.

6: There's really nothing unique about me.
A number divisible by 6 must also be divisible by 2 and by 3.

8: Throw away the thousands and above, don't hesitate.
The last three digits must be divisible by 8.

9: I'm similar to 3, but a little more refined.
The sum of the digits must add up to 9.

After reading the poem, every gifted child wants to know if there's a rule for the 7's. There is, but it's almost easier to do the division. For those who are overcome by curiosity, here's the rule: Multiply the ones digit by 2, then subtract it from the remaining digits. Continue doing this until the number is small enough to recognize whether it's a multiple of 7. Try, for example, 5492. 549 – 4 (ones digit x 2) = 545. 54 – 10 (new ones digit x 2) = 44. Since 44 isn't divisible by 7, 5492 isn't either. The reason this works is because it results in subtracting multiples of 21 (a multiple of 7) from the initial number. Somehow I could not compose a brief rhyme for this one!

TRANSFORMERS

These puzzles are also known as "Word Arithmetic." I haven't seen any online, but they are popular in puzzle magazines. As the name implies, most sources produce puzzles in which the quotient, divisor, dividend, and 10-letter solution are all words or phrases. I've chosen not to use words since these puzzles are for the gifted population. Having worked with gifted children, I'm certain that some of them would spend hours trying to find the solution by rearranging the letters instead of doing the math.

To explore the inverse nature of multiplication and division, ask the students if they can transform the puzzle into a multiplication problem. Where would the divisor and quotient go? What about the remainder?

TRISQUARES

I literally dreamed up these puzzles. I'd been spending a lot of time making puzzles, to the point that I was even dreaming about them. One night, I dreamed I was solving a puzzle. As I was waking up, I realized that I might be able to make a new type of puzzle like the one in my dream, so I made sure that I remembered how it worked. I had to do a little tweaking when I tested it out during the day, but TriSquares were the end result.

As an extension activity, have the students calculate the Least Common Multiple of the digits 1 through 9. Would they have to multiply all nine digits? What shortcuts could they use? How will they know when they've found the very lowest number that's divisible by all nine digits? Will this Least Common Multiple be evenly divisible by each of the products given in each puzzle? Why or why not?