

TABLE OF CONTENTS

Introduction	1
Introductory Lesson: What Is a Mathematician?	4
Unit 1: Systems of Counting	9
Lesson 1.1: Explore Different Counting Systems	10
Lesson 1.2: Apply Binary Counting	15
Lesson 1.3: Project: Systems of Counting Around the World	20
Unit 2: Order of Operations	31
Lesson 2.1: Explore the Order of Operations	32
Lesson 2.2: Apply the Order of Operations	36
Lesson 2.3: Project: Reflect on the Order of Operations	41
Unit 3: Patterns	47
Lesson 3.1: Explore Patterns	48
Lesson 3.2: Apply Patterns to the Golden Spiral	52
Lesson 3.3: Project: Analyse Patterns in Art	56
Unit 4: Time and Measurement	63
Lesson 4.1: Explore Time on Planets	64
Lesson 4.2: Apply Solar Maths	68
Lesson 4.3: Project: Design Solar Models	72
Unit 5: Graphing, Data and Charts in Algebra	77
Lesson 5.1: Explore Linear Data, Charts and Graphs	78
Lesson 5.2: Apply Nonlinear Data, Charts and Graphs	82
Lesson 5.3: Project: Create a Business Plan	87

THINKING LIKE A MATHEMATICIAN

Unit 6: Geometry	95
Lesson 6.1: Explore Perimeter and Area	96
Lesson 6.2: Apply Properties of Quadrilaterals	101
Lesson 6.3: Project: Analyse Famous Architecture	106
Unit 7: Data Analysis and Statistics	113
Lesson 7.1: Explore Tidal Data and Statistics	114
Lesson 7.2: Apply Data Analysis	118
Lesson 7.3: Project: Shipwrecked!	122
Final Project: Designing a Garden	127
Answer Key	133
About the Authors	141

INTRODUCTION

PURPOSE

Maths is everywhere, and it is exciting to introduce students to real-world applications of the concepts they study in school each day. *Thinking Like a Mathematician* provides opportunities to connect your maths content to diverse mathematicians, introductory activities and real-world applications.

This book invites students to engage with seven units of study and a culminating problem-based assessment. There are multiple ways to use this text. The units can be taught sequentially, resulting in a thorough exploration of the field of mathematics, or educators can choose to teach the units, lessons and projects in isolation to complement an existing curriculum. We hope that *Thinking Like a Mathematician* guides your students' learning in rich and meaningful ways.

ORGANISATION OF THE BOOK

Lesson 1 (Explore) and Lesson 2 (Apply) of each unit are designed for you to instruct students on new material. These activities might be most meaningful for students who have a little background knowledge prior to beginning, but they are certainly full of teachable moments for you to guide student learning. Each lesson's activities require students to think about maths in different ways, but please reflect on your individual students' needs as you make adjustments.

Each unit concludes with a research project in Lesson 3. Students could investigate each topic as partners or as individuals in these highly adaptable projects. The projects provide students structured guidance but also allow choices on what topic to explore. As students learn to research, you should guide their time management. Partnerships with your school librarian and technology staff will help build students' research skills. The rubrics for the projects are included at the end of each unit.

THINKING LIKE A MATHEMATICIAN

As students begin to work in groups, they will need your direct instruction on the maths components, but also the components required to be helpful, contributing teammates. These interpersonal skills are the foundation for the projects in this book.

STUDENT MATHS NOTEBOOKS

Writing and reflecting on maths concepts are important skills for students to develop. Many of the lessons make reference to student maths notebooks, in which students can answer Anticipatory Set and Exit Ticket questions. Teachers should provide spirals or binders for students to use. Students should also answer questions in the maths notebook if there is not a handout to use. To make these notebooks more dynamic, students may pick pictures of mathematicians (and themselves) for the notebook cover. Students can build these maths notebooks to use as a reference throughout the year.

REFLECTION

As they explore content and interpersonal relationships, students need time to reflect upon their progress. They should explore their background knowledge and new learning, consider which products might represent their learning best, decide how to analyse and tackle obstacles, and learn about their own time management skills. Included with Lesson 1.3 is a reflection page, which is designed for students to visit multiple times throughout each of the projects, not just once at the end. Learning time is precious, and a quick conversation can be more meaningful than a lengthy analysis.

The student reflection helps students consider their effort, teamwork and product. High-ability students may be used to coasting through assigned classwork independently. These rigorous projects require them to investigate real-world problems in challenging ways. Inevitably, students will confront obstacles, allowing for discussion and teachable moments.

Introduction

THINKING LIKE SERIES

This book is one in a series, developed in conjunction with the Center for Gifted Education at William & Mary, intended to develop process skills in various learning areas and enhance discipline-specific thinking and habits of mind through hands-on activities. Each book in the series focuses on a specific discipline and year level:

- In *Thinking Like a Geographer*, students in Years 2–3 develop and practise geography skills, such as reading and creating maps, graphs and charts; examine primary and secondary sources; and think spatially on a variety of scales
- In *Thinking Like a Mathematician*, students in Years 3–4 engage in exploration activities, complete mathematical challenges and then apply what they have learned by making real-world connections.
- In *Thinking Like an Engineer*, students in Years 4–5 complete design challenges, visit with an engineer and investigate real-world problems to plan feasible engineering solutions.
- In *Thinking Like a Scientist*, students in Years 5–6 use inquiry-based investigations to explore what scientists do, engage in critical thinking, learn about scientific tools and research, and examine careers in scientific fields.

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INTRODUCTORY LESSON

WHAT IS A MATHEMATICIAN?

What is a mathematician? What do mathematicians look like? What do they do every day? Where do they work? These are all valuable questions to pose to your students to understand their background and misconceptions.

As you begin your exploration of maths concepts with high-ability students, you may choose to organise their thinking using a Frayer model. This organiser guides students' learning as they explore a new topic. In this case, students will explore the concept, "What is a mathematician?" Students may visualise many different people when they think of "mathematician". This initial conversation can act as a pre-assessment for future units. Listen carefully and be ready to dispel myths or misunderstandings.

What Is a Mathematician?

RESOURCES AND MATERIALS

- Introductory Lesson Frayer Model
- Large class Frayer model
- Student maths notebooks
- Bulletin board with pictures and identifying information of modern mathematicians (assemble in advance; consider using a diverse range of people, such as Julia Robinson, Srinivasa Ramanujan, Katherine Johnson and Dorothy Vaughn, or local mathematicians from universities in your area)
- Photo booth with maths tools/props (e.g. rulers, tape measures, protractors, compasses, an abacus, thermometers, calculators, set squares, slide rules, scales, pan balances, a telescope, lab tools and Galileo's thermometer)

ESTIMATED TIME

45 minutes

OBJECTIVES

In this lesson, students will:

- use a Frayer model to brainstorm characteristics of a mathematician
- discuss what attributes make for a successful mathematician
- begin to envision themselves as mathematicians.

PRIOR KNOWLEDGE

Students will need to know how to share ideas thoughtfully and politely, especially when disagreeing with peers.

INSTRUCTIONAL SEQUENCE

1. Distribute Introductory Lesson Frayer Model and pose the following question: *What is a mathematician?* Give students time to complete the hand-out and discuss with a partner.
2. Record students' suggestions on a large class Frayer model. As students make suggestions, there may be disagreement on whether the ideas make sense. For the purposes of brainstorming, record each of the ideas.
3. Review each quadrant in the Frayer model, posing questions to guide students' discussion. Consider questions like:
 - What do mathematicians do?
 - How do they act?
 - What are their personalities like?
 - What jobs/hobbies do they have? What jobs/hobbies do they not have?
 - Is there an opposite of a mathematician?

THINKING LIKE A MATHEMATICIAN

4. At the end of the conversation, ask students to choose the best 5–7 ideas in each quadrant of the Frayer model. Ask: *What makes these ideas the best? How do they help us understand what a mathematician is?*
5. Present the mathematician bulletin board to your students (see Resources and Materials). This board can be used throughout the following units, with students adding information about other mathematicians as they research.
6. Point out the diversity represented on the mathematician bulletin board before announcing that there are some mathematicians very close by whom you would like to add ... the students! Set up a photo booth with maths tools and props to take a photo of each student to add to the board.
7. Discuss students' maths notebooks. Students may select pictures of mathematicians (and themselves) for the notebook cover. Explain that they will build their maths notebooks to use as a reference throughout the year.

EXTENSION ACTIVITIES

- Have students write a letter to a mathematician in the community posing some of the questions from the discussion.
- Have students create a chart connecting the character traits of mathematicians to books that they have read.

ASSESSMENT OBSERVATION

Students should respond to the question “What is a mathematician?” in their maths notebooks.

UNIT 1

SYSTEMS OF COUNTING

RATIONALE

This unit shows students different ways of counting. Students will learn that the base-10 numbering system is not always used. The binary system will be introduced, and students will learn that computers use the binary system. This will solidify students' understanding of place value and how the number system works. This unit will also introduce basic computer concepts to students.

PLAN

In Lesson 1.1, students will explore the base-10 numbering system. Students will also explore other base numbering systems. In Lesson 1.2, students will apply different numbering systems to write binary code. In Lesson 1.3, students will discover different ways of counting around the world as they develop and present a project.

LESSON 1.1

EXPLORE DIFFERENT COUNTING SYSTEMS

RESOURCES AND MATERIALS

- Lesson 1.1 Base-4 Counting
- Three stackable cups with the numerals 0, 1, 2 and 3 written around each cup lip (per group; see Figure 1 for reference)
- Student maths notebooks

ESTIMATED TIME

40–45 minutes

OBJECTIVES

In this lesson, students will:

- discover a pattern from a base-4 numbering system
- understand the differences between numbering systems
- compare and contrast the base-10 numbering system with a base-4 numbering system.

PRIOR KNOWLEDGE

Students will need a basic understanding of a base-10 number system. They should also understand place value and how to add and subtract three-digit numbers.