ZEROING IN ON NUMBER AND OPERATIONS
Key Ideas and Common Misconceptions, Grades Pre-K–K

Linda Dacey and Anne Collins

CONTENTS

Introduction

The foundation for number and operations is built at the pre-K–K level. Too often, early childhood education does not place enough emphasis on the learning of mathematics, and when it does, the math instruction is not always taught in a developmentally appropriate manner. Young children should have the opportunity to explore mathematics through their natural play and through structured games and activities that meet their individual learning needs.

The National Governors Association and the Council of Chief State School Officers (2010) have recommended a curriculum that is focused with an emphasis on key topics. In prekindergarten and kindergarten, we focus on the key ideas that are essential for success at these levels:

- counting by rote, that is, saying the sequence of counting numbers in order
- counting rationally (with one-to-one correspondence and cardinality)
- subitizing, or recognizing sets
- counting by tens and ones
- making connections among representations of numbers
- comparing and ordering sets and numbers
- composing and decomposing numbers
- modeling addition as the joining of sets
- modeling subtraction as the separating of a set

The thirty modules in this flipchart are designed to engage all students in mathematical learning that develops conceptual understanding, addresses common misconceptions, and builds key ideas essential to future learning. The modules are research based and can be used to support a wide range of learners. Though Response to Intervention (RTI) is not often formalized at these levels, its philosophy of a well-integrated instructional system that addresses students' academic needs is embraced. These modules offer suggestions and resources for teachers seeking material for students identified as most likely to benefit from more support as well as enrichment activities and challenges for all students.

Following the recommendations of the National Council of Teachers of Mathematics (Fuson, Clements, and Beckmann 2010a, 2010b), we have organized these modules into three sections: Numbers, Relationships, and Addition and Subtraction. Each module begins with the identification of its **Mathematical Focus** and the **Potential Challenges and Misconceptions** associated with those ideas. **In the Classroom** then suggests instructional strategies and specific activities to implement with your students. **Meeting Individual Needs** offers ideas for adjusting the activities to reach a broader range of learners. Many activities are supported by a reproducible (located in the appendix), and **References/Further Reading** provides resources for enriching your knowledge of the topic and gathering more ideas.

Pre-K–K teachers work with children representing a wide range of developmental readiness. We have covered the full range of possibilities in these pages so you will be able to address multiple levels of learning, regardless of the specific ages of your students. Feel free to adjust the modules to best match your learners. You could modify the size of the numbers or the expectations for recording to make an activity more appropriate for your students.

REFERENCES/FURTHER READING

Fuson, Karen C., Douglas H. Clements, and Sybilla Beckmann. 2010a. *Focus in Kindergarten: Teaching with Curriculum Focal Points*. Reston, VA: National Council of Teachers of Mathematics.

———. 2010b. *Focus in Prekindergarten: Teaching with Curriculum Focal Points*. Reston, VA: National Council of Teachers of Mathematics.

National Governors Association (NGA) Center for Best Practices and Council of Chief State School Officers (CCSSO). 2010. Common Core State Standards. http://www.corestandards.org/the-standards.

One, Two, Three

Mathematical Focus

- Count to three (or four or five) using one-to-one correspondence.
- Count up to three (or four or five) objects to find how many.

Potential Challenges and Misconceptions

The number three can provide a challenge to our youngest students. Examples of one and two are commonplace in their lives (one head, one nose, two hands, two feet, two eyes), while examples of three are less common.

Though many young children can say the names of the numbers to three, or five, or even ten, this is only rote counting. Counting also includes the ability to say exactly one number name for each object counted. Even children who can count with this one-to-one correspondence may not realize that the last number they say is the number of items in the group. This aspect of counting is known as *cardinality*.

In the Classroom

One preschool teacher places three carpet samples in a row. She invites the two- and three-year-old children to count aloud with her as she points to the carpet pieces one at a time while saying, "One, two, three." She asks, "How many carpet pieces do we have?" Next she chooses three volunteers and has the first child stand on the first square and say, "One." The second and third children follow, saying, "Two" and "Three." Then the teacher asks, "How many carpet pieces do we have? How many children are standing on these pieces of carpet?" Note that some learners may focus on the fact that there is one child on each piece of carpet and reply, "One," and others may need to count again to be sure. This is fine and just indicates that they need more opportunities to count to three. You can add more carpet pieces when the children are ready or when working with learners who are four or five years old.

Games also provide a good context for counting. For example, lead a game of "Teacher, May I?" and have the children take one to five baby or giant steps, counting aloud as they do so.

Meeting Individual Needs

The suggested activities are multisensory, which supports a variety of learners. Using a context with which children are familiar can also be beneficial. Several stories such as *Goldilocks and the Three Bears* and *The Three Little Pigs* bring attention to the number three. To provide children greater familiarity with three, read these stories aloud and have the children dramatize them.

It is important for children to develop one-to-one correspondence and cardinality with small numbers first. Once they master the numbers up to three, you can provide the challenge of working with greater numbers.

REFERENCES/FURTHER READING

Anderson, Ann, Jim Anderson, and Carolyn Thauberger. 2008. "Mathematics Learning and Teaching in the Early Years." In *Contemporary Perspectives on Mathematics in Early Childhood Education*, ed. Olivia N. Saracho and Bernard Spodek, 95–132. New York: Information Age.

Sarnecka, Barbara W., Valentina G. Kamenskaya, Yuko Yamana, Tamiko Ogura, and Yulia B. Yudovina. 2007. "From Grammatical Number to Exact Numbers: Early Meanings of 'One,' 'Two,' and 'Three' in English, Russian, and Japanese." *Cognitive Psychology* 55 (2): 136–68.

Mathematical Focus

- Say the names of the numbers in order.
- Connect the counting number sequence to the idea that each number is one more than the previous number.

Potential Challenges and Misconceptions

Memorizing the names of the counting numbers can be challenging for many children, especially if rote counting (saying the number names in order) is not incorporated into routines at home or if children are English language learners. One of the many functions of music is that it makes things easier to remember (Pound and Harrison 2003). When combined with finger actions or other dramatizations, songs can also support one-to-one correspondence and cardinality, or counting with meaning. Do not be surprised if children who are successful when they are singing experience difficulty when saying the number names without the support of a familiar song or rhyme.

In the Classroom

There are a variety of common songs and rhymes that involve the counting sequence. Two- and three-year-olds enjoy marching around while reciting or singing "One, Two, Buckle My Shoe," "Five Little Monkeys," and "This Old Man." You can also introduce songs for the children to dramatize. The process of doing so will help them make connections between number names and amounts. Consider the following two songs (sung to the tune of "Five Little Sea Shells," which can be heard at http://www.youtube.com/watch?v=42ilv3wc338) that are easily extended to five or ten. Counting backward is more challenging and generally reserved for older learners.

Try the following when introducing such songs:

- Have the children practice singing the words first.
- Have the children practice singing the song as they use their fingers to show the action.
- Ask for suggestions about how a small group could dramatize the song.
- Invite volunteers to dramatize the action as the remaining children continue to hold up the correct number of fingers as they sing the song.

Meeting Individual Needs

You can easily adapt these songs to meet a variety of interests. Racing cars may be more engaging than twinkling stars. You can also make connections to your classroom areas, for example, by changing "Ten in the Bed" to "Ten on the Rug." You can start or end songs at three, five, or ten to meet different levels of readiness. Encourage children who speak languages other than English to teach the class songs in their languages.

REFERENCE/FURTHER READING

Pound, Linda, and Chris Harrison. 2003. *Supporting Musical Development in the Early Years*. Buckingham, UK: Open University Press.

One little doggie sitting on a chair.
One more doggie comes to be there.
Two little doggies sitting on chairs.
One more doggie comes to be there.
Three little doggies sitting on chairs.
Don't you wish that you could be there?

Three twinkling stars lighting up the sky.
One gets covered by a cloud coming by.
Two twinkling stars lighting up the sky.
One gets covered by a cloud coming by.
One twinkling star lighting up the sky.
The cloud comes by and then there are none.

All About Five

Mathematical Focus
- Connect number names, symbols, and quantities up to five.
- Recognize amounts of one through five without counting (subitize).

Potential Challenges and Misconceptions

Number sense is more than counting. Over time, children gain the ability to recognize the number of objects in a small group without even counting them. With many opportunities to count, two- and three-year-olds may recognize a group of one, two, or three items. Four- and five-year-olds can learn to recognize greater numbers as well, especially if they are arranged in a familiar pattern, such as the way five is represented on dice. This ability is known as *subitizing*.

Some children only subitize the numbers one and two, which will lead to later difficulties. Knowing a number such as five well allows it to serve as a benchmark, that is, a number to which other numbers can be compared, an idea critical to ordering numbers. Children of ages four and five can use their recognition of the number five to build a sense of the numbers six through ten.

In the Classroom

Many children have a visual image of five, as they know they have five fingers on each hand. One preschool teacher uses finger puppets to connect the process of counting to five to that of finding the total quantity of five. As the teacher places each puppet on a child's hand, the child counts it aloud. When asked how many puppets there are in all, many children count again from one, rather than just reply, "Five." This teacher makes a point to reinforce the notion of cardinality by saying, "Yes, the last number you said tells us there are five puppets. Can you wiggle all puppets at once?"

Place sticky dots on paper plates to provide a quick representation of numbers. Stick one on a plate, two on another, and so forth. You can ask younger prekindergarten children to place a counter on each dot to count the dots; to make a set of counters on another plate to match the number of dots on a given plate; or to recognize the plates for one to three when flashed quickly. Older pre-K and K learners can match different arrangements of the same number of dots; recognize sets of four and five; and match the number of dots to number symbols.

Five-frames offer a more structured organization of the numbers one through five. An empty frame can be used as a frame upon which children can build numbers or as a model for zero. Make copies of the *Five-Frames* reproducible on page A1 in the appendix. Cut out the frames and laminate them. The frames can be used in ways similar to the dot plates, but they better support understanding of relationships among numbers. For example, they can help learners recognize that four is one less than five (see figure).
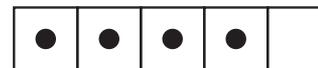
Meeting Individual Needs

The symbol for two is often confused with the symbol for five. There are a variety of ways to involve kinesthetic learning in the recognition of symbols that can help children better differentiate these numbers. Examples include pretending to write large symbols in the air and tracing sandpaper symbols with their fingers.

REFERENCE/FURTHER READING
Novakowski, Janice. 2007. "Developing 'Five-ness' in Kindergarten." *Teaching Children Mathematics* 14 (4): 226–31.

Counting Routines

Mathematical Focus
- Count by rote.
- Count rationally.

Potential Challenges and Misconceptions

Counting is much more complex than many adults realize. Children must know the correct sequence of the number names; count one and only one object for each number name they say; know that the last number they say identifies the number of objects in the group (cardinality); know that the cardinality of a group does not change if the objects are rearranged (stability); and make connections between number names, quantities, and symbols.

Too often adults assume that children who exhibit these abilities with numbers less than five (or ten) will also do so with greater numbers. This is often not the case. It takes time and considerable practice for these concepts and skills to develop. We must support routines that involve counting and take advantage of the many opportunities available to integrate number activities throughout the day.

In the Classroom

There are many opportunities for children to practice counting during daily activities, for example, climbing up or down stairs, taking attendance, passing out paintbrushes, and keeping track of the number of questions allowed to be asked at morning meetings. You can also create a variety of routines that support both rote counting and counting with meaning (rational counting). For example:

- *Number Tray*: Each day, place some counters on a lunch tray. Sometime during the day, each child should count to find the quantity on the tray. Depending on ability, the child should then either whisper his or her answer to you or write the number on a slip of paper and hand it to you.
- *Estimate It*: When outside, ask questions such as *How many steps do you think it is to that tree?* Then have the children count each step as they walk to the tree.

- *Clothes Counts*: While sitting in the rug area, ask, *How many shoes (buttons, stripes, pockets) are you wearing today?* After each child counts to decide, have three children demonstrate their counts.
- *Tower Fall*: Have children place one block on top of another, counting as they do so, until the stack falls down.
- *Flash*: Cover a plate of counters with a cloth. Quickly remove and replace the cloth. Ask the children to estimate how many counters they saw. Then choose a volunteer to count to check.

Meeting Individual Needs

English language learners may be able to count to greater numbers in their languages but lack knowledge of the number sequence in English. While they learn the new number names, have these children occasionally count in their languages as well. While they do so, you can check their understanding of one-to-one correspondence, cardinality, and stability. Also, encourage choral counting so that the voices of others can prompt ELLs to remember the correct names and sequence.

REFERENCES/FURTHER READING

Galizio, Carolyn, Julia Stoll, and Pamela Hutchins. 2009. "'We Need a Way to Get to the Other Side!': Exploring the Possibilities for Learning in Natural Spaces." *Young Children* 64 (4): 42–48.

Kalifatidou, Eleftheria R. 2008. "Design of Activities on Numerical Representations Based on Cognitive Research." *Teaching Children Mathematics* 14 (6): 355–60.

I Spy

Mathematical Focus

- Relate counting and cardinality.
- Recognize and use number names.
- Realize that the number of a set does not change when the items are rearranged (stability).

Potential Challenges and Misconceptions

Two- and three-year-old children love to recite the number names, but most lack understanding of their meaning. Sometimes they count in the correct sequence and other times they do not. Children often continue saying the number names after all the objects have been counted. At a restaurant, a two-year-old might be given four crayons and a placemat to color. She might say, "Watch me count these," and count from one to ten, pointing to various crayons as she does so, stopping only because ten is the last number name she knows.

Her four-year-old brother might look over and without counting say, "No, there are four." Yet this same boy might be given seven animal crackers, skip one while he counts them, and report that he has six. Both of these children are developing their number sense as they continue to count sets and develop facility with all aspects of counting.

In the Classroom

I Spy is a game in which young children are challenged to identify the mystery set that is described. An adult begins by putting different amounts of objects on various trays and placing the trays around the room in visible locations. The objects may include a variety of counters, including but not limited to small cars, balls, blocks, crayons, pencils, and large buttons.

For two- and three-year-olds, the game begins with the teacher choosing from one to three objects. The teacher might say, "I spy a set of two. Who can tell me what is in the set I see?"

The children look around a small area of the room to see if they can identify a set of two. A child might say, "I see two balls."

The teacher continues by stating, "I spy a set of three. Who can tell me what is in the set I see?"

Older children can assume the role of teacher, saying, "I spy a set of _____. Who can tell me what is in the set I see?"

To assess whether children understand that rearranging the objects does not change their quantity, move the objects on the tray just "spied" and provide the same number clue. Note those children who understand the stability of numbers and have the memory skills to identify the set without counting.

Meeting Individual Needs

You can individualize the number of objects included in the game *I Spy* for various children. Give those children who can identify only a set of up to three items the time they need to develop their confidence before increasing the number of objects, regardless of their age.

REFERENCE/FURTHER READING

Van de Walle, John A., Karen S. Karp, and Jennifer M. Bay-Williams. 2010. *Elementary and Middle School Mathematics: Teaching Developmentally.* 7th ed. New York: Pearson Education.

Which One?

Mathematical Focus

- Count objects that are immovable.
- Use the process of elimination.

Potential Challenges and Misconceptions

It is a challenge for children to keep track of items they are counting. They may skip an item or count it more than once. When they use counters, young students can move each one off to one side after counting it. But keeping track visually, when they cannot move objects, is more difficult. Some children may want to make a mark on each item they count in a picture to help them keep track.

In the Classroom

Using clues to identify the correct picture gives children opportunities to count objects that cannot be moved. The *Find It* reproducible on page A2 in the appendix provides pictures for you to copy. The pictures can be used with a wide variety of learners, so you may want to copy them on heavy stock and use them at different times during the year. For example:

- With a small group of two- and three-year-old children, show them just one of the pictures in each row at a time and ask them counting questions. For instance, show a picture of one of the plants and ask, *How many flowers does this plant have? How many leaves does this plant have? How many stripes do you see on the pot?* Help the children count each of the items you specify.
- Show some three-year-olds and most four-year-olds all three of the pictures of the plants and ask, *What is the same about these plants? What is different?* Then say, *I have a plant that has exactly three leaves. Which picture shows this plant?* Help the children check the number of leaves on each plant. Similarly, you could identify this same plant with the clue *I have a plant with one flower* or *I have a plant in a pot with two dark stripes.* You can use the pictures of houses and monsters in the same manner.
- For learners ready for a greater challenge, you can provide two clues they must use to identify a plant. For example, tell the children that they are going to use clues to find the picture of Mr. Chen's plant. Give the first clue, *Mr. Chen's plant has four leaves*, and record a 4 on chart paper or the board. Help the children check each picture and recognize that they can eliminate one of the pictures. Then follow the same process for the final clue: *Mr. Chen's plant has one flower.* Once the children have identified the correct plant, have the children look at it while you review the clues: *four leaves*, *one flower*. Say, *This picture meets all of the clues.*

For the house and the monster pictures, you can provide students one clue to identify the correct picture or give them the following sets of clues.

In which house does Krista live?	Which monster did Rusty draw?
• Her house has five windows. • Her house has two trees in front of it.	• His monster has three eyes. • His monster has five legs. • His monster has two hairs.

Meeting Individual Needs

To help children keep track of pictures they have eliminated, cut out the pictures in each row so that students can turn them over when they don't match a clue. Alternatively, make copies of the reproducible on copy paper and have children cross off any picture that does not meet a clue.

Make the clues more complex to offer a greater challenge. For example, change *There are four leaves* to *There are two more leaves than flowers.*

REFERENCE/FURTHER READING

Sarama, Julie, and Douglas H. Clements. 2007. "Early Math: How Children Problem Solve: Helping Children Use Problem-Solving Strategies in the Classroom." *Early Childhood Today* 21 (7): 16–19.

From Five to Ten

Mathematical Focus
- Anchor numbers to five and ten.
- Count on from five.
- Recognize numbers represented on a ten-frame.

Potential Challenges and Misconceptions

The likelihood that children will make counting errors increases with greater quantities. As the number of items to be counted increases, children might skip a number name, neglect an object, or count an item more than once. Arrangement also matters. For example, it is easier to count objects in a row than when they appear in a random organization. Placing items in groups of five or ten helps learners keep track of what they are counting.

Organizing numbers in five-frames (see "All About Five") helps students count with accuracy and relate numbers to the benchmark number five. Two rows of five squares create a ten-frame, which is introduced in this activity.

In the Classroom

Model counting a group of six counters by placing the first five counters in an empty five-frame (see the *Five-Frames* reproducible on page A1 in the appendix) and one more counter beside the frame, as shown (see figure). Then ask the children to do the same. Using the five-frame helps children see six as one more than five. Have children tell the number of counters and match the set to the number symbol. Follow the same procedure for the numbers seven through nine.
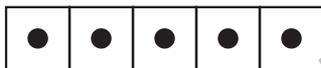
Children can also relate these numbers to their fingers; that is, they can show five on one hand and one to four more on the other hand. Over time, some children will be able to recognize the five and *count on*, saying, "six, seven, eight, nine."

Kindergarten students can be introduced to the ten-frames, which are 2-by-5 arrays used to model the numbers one through ten in a way that anchors them to five and ten. Make copies of the *Ten-Frames* reproducible on pages A3–A6 in the appendix on heavy stock, cut out the frames, and give a set to each child. The frame for eight, for example, provides a visual model of eight as three more than five and two less than ten (see figure). This visual model supports eventual connections to $5 + 3 = 8$ and $10 - 2 = 8$. Once you've had students build the numbers in order, call out numbers in a random sequence and have the children build them on empty frames.
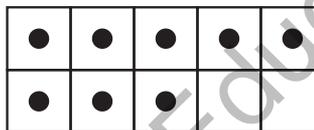
These frames can be used in a variety of ways. For example:

- Have children arrange the frames in order, with or without your support, and then ask them to identify frames as you call out random numbers.
- Give pairs of children two sets (or partial sets) of ten-frames and have the children match the frames that show the same numbers.
- Have children work in pairs with one set of frames organized in order. One child closes his or her eyes while the other removes one of the frames. The first child then identifies the one that is missing.
- Hold up a frame and have the children tell the number they see.

Meeting Individual Needs

While there is great value in using the frames, children still need many opportunities to count objects without using these tools. Have children count objects in lines, arrays, circles, and then random arrangements to assess their ability to keep track. Make a copy of the *Counting Assessment Form* reproducible on page A7 in the appendix and note each child's ability to work with different numbers. Make and complete another copy later in the year to document growth over time.

REFERENCE/FURTHER READING

Losq, Christine. 2005. "Number Concepts and Special Needs Students: The Power of Ten-Frame Tiles." *Teaching Children Mathematics* 11 (6): 310–15.