How to Use Multiple-Choice Tests to Improve Maths Instruction
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Multiple-choice testing is an educational reality for students and teachers. Rather than continue to complain about how these tests can adversely affect teaching and learning, we thought it better to turn the testing situation on its head – that is, to take full advantage of all that multiple-choice testing can offer. The purpose of Beyond the Bubble is to do just that: show teachers how to get more from multiple-choice tests. By asking students just a few carefully chosen questions, teachers can gain valuable insight into students’ mathematical thinking.

Many schools rely on multiple-choice testing to assess students’ maths progress. The assumption is that if a student marks a correct answer – if they fill in the right bubble – that student is proficient in the corresponding skill or objective. However, a correct answer can often mask fragile knowledge or misconceptions or it may have been just a lucky guess. (There are many examples of this throughout the book.) The inverse is also true. If a student marks an incorrect answer – if they fill in the wrong bubble – that student is considered to be in need of remediation. But the student may have just misread the problem or made a mistake when selecting an answer. Both “correct” and “incorrect” answers reveal little about what a student truly does or doesn’t understand. Consequently, instructional decisions based on this testing information may be misguided. Again, taking just a few moments to probe students’ thinking can provide valuable insight leading to more effective instruction for all students.

Using typical multiple-choice questions often found on Year 4 and 5 assessments and in test-prep materials, we asked hundreds of students to explain their answer choices in writing and verbally. We found that both correct and incorrect multiple-choice responses often painted an inaccurate or incomplete picture of students’ mathematical understanding. For example, we assumed students who answered questions correctly would consistently show strong understanding and demonstrate logical thinking, but they just as often showed partial understanding, confusion or no understanding at all. We were surprised to find the same was also true for students who marked incorrect responses. But using these test questions and probing with a few additional questions allowed us to get “beyond the bubble” – suddenly we were using the questions to our (and our students’) advantage, uncovering understanding and misconceptions, which, in turn, allowed us to make more effective instructional decisions as we considered what our students needed next.

Beyond the Bubble is divided into five strands: number, measurement, algebra, geometry and probability. There are six problems per strand. Each problem begins with a brief overview of the test question’s objective, followed by the sample test question, typical student strategies used to solve the problem, conversation starters, actual student work, student-teacher conversations along with teacher insights and suggestions for instructional strategies that should help advance individual students’ learning. Many of the “Informed Instructional Suggestions” sections include ideas for differentiating instruction and grouping students with similar needs. Reassessment questions are also provided.

At the end of the book there is a general list of questions for you to use to start conversations with your students. We’ve found that posting these questions on the back wall of your classroom provides a quick and easy way to use them with students when having conversations throughout the school day. Also included are reproducibles of all sample questions and a general list of teaching resources.

You may notice that some students’ work appears more frequently throughout the book. These students include Penny, Lilli, Suri, Kylie, Darren and Eddie. You may find it worthwhile to study the work of each of these students and discuss your insights with colleagues. What questions would you ask the student? What
do you think the student knows? Where do you think gaps might be? How do you know? How could you find out? Such reflection will help you prepare for having meaningful conversations with your students that you can use to inform your instruction.

Some Dos and Don’ts

• Do take the time to ask questions and listen carefully to students’ responses. They will provide you with valuable opportunities to understand and appreciate your students’ strengths and weaknesses.

• Don’t rely on a single multiple-choice response alone. It may mask true understanding or misunderstanding, making purposeful instruction difficult.

• Do discuss what you find out with colleagues. Talk about surprises, victories and methods to engage students and help them move forward with understanding.

• Don’t be afraid to follow a child’s lead. You may not understand their thinking initially, but by listening carefully with an open mind, you may discover brilliance in ways you’ve never before considered.

• Do reflect on our examples and see if you can find similar outcomes in your class. The more connections you make to your children, the more comfortable you will be with engaging students in meaningful mathematical conversations, ultimately improving your instruction and children’s learning.

• Do keep asking good questions that uncover students’ learning and understanding, providing you with valuable insights. Children deserve our attention and our best instructional decisions.

As educators, the more information we can gather, the better pedagogical decisions we can make for our students. We wrote *Beyond the Bubble* for all educators who want better, more focused mathematics instruction for their students. This includes teachers, administrators and pre-service teachers. The results of our work with students provide the basis for excellent in-service discussions or professional learning community (PLC) planning and conversations. When instructional decision makers, both teachers and administrators, examine students’ written and verbal explanations, differentiation becomes quicker, easier and more targeted. It is our hope that *Beyond the Bubble* will be used as a tool for insightful, engaging mathematics instruction for you and your students.
Problem One

Overview

Students must apply their number sense, their ability to count by tens and their understanding of place value and of number lines to answer this typical multiple-choice question.

Sample Problem

In which box on the number line does the number 190 belong?

A. Box A  
B. Box B  
C. Box C  
D. Box D

Explain your thinking.

Possible Student Solution Strategies

• Students use the numbers provided to find a pattern of counting by tens and use this information to locate where 190 belongs.
• Students count by an incorrect interval.
• Students fail to use the provided numbers correctly to determine where 190 belongs.

Conversation Starters

• How did the information provided help you solve this problem?
• What is another way you could solve this problem?
• How do you know your answer is reasonable?
• How can you prove your answer?
• What pattern did you see that helped you solve this problem? Describe it.
**Student Work Sample: Preston**

In which box on the number line does the number 190 belong?

```
A  170  B  180  C  200  D  230
```

- Box A
- Box B
- Box C
- Box D

Explain your thinking.

It's not "A" because 190 is more than 170.
It’s not "B" because 210 is more than 190. It’s not "C" because 240 is more than 190.
It’s "D" because their counting by 10s and 180+10=190.

---

**A Conversation with Preston**

**T:** How did the information provided help you solve this problem?

**Preston:** I noticed that the number 170 was the first number there. I also saw that 230 was there and a bunch of lines. I knew that the lines couldn’t be counting by ones because there were not enough lines between 170 and 230, so I tried counting by tens and it worked! So then I filled in the missing numbers. That made it pretty easy to find out where 190 was supposed to go.

**T:** I appreciate your clear written explanation and how you were able to explain your thinking aloud as well.

**Teacher Insights**

**T:** Preston was able to apply his understanding and reasoning to efficiently solve this problem.

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**Informed Instructional Suggestions**

Preston is ready to explore number lines with more complex patterns and larger numbers. As Preston becomes increasingly adept, he can work with number lines that include fractions, decimals, per cent and negative numbers. It would also be appropriate to have Preston create his own number lines with missing numbers for other students to complete.
Student Work Sample: Sumita

Name __________________________ Date ______________________

In which box on the number line does the number 190 belong?

A B C D

A. Box A
B. Box B
C. Box C
D. Box D

Explain your thinking.

**it's B because I started from 230 and at B I got 190.**

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<td><strong>T:</strong> I see that you were able to select an answer. What could you have included on your paper to prove your answer makes sense?</td>
<td><strong>T:</strong> Sumita is fluent with the skills involved in solving this problem. She was able to explain why her solution made sense in two ways.</td>
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<td><strong>Sumita:</strong> What I did was start at 230 and I guessed and counted backward by tens until I got to 190, which was B. Then I started with 170 and counted up by tens and landed on B again. I was pretty sure I was right. But I guess I could have written the numbers on the number line, but I did it two ways, so I know I'm right.</td>
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<td><strong>T:</strong> I am curious about why you decided to count backward first.</td>
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<td><strong>Sumita:</strong> It's more interesting to me to do it that way. Then I can count the other way to make sure the backwards way works.</td>
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Informed Instructional Suggestions

Like Preston, Sumita is able to easily make sense of problems at this level. Sumita is ready for the same activities we suggested for Preston.
Reassessment

1. Use a similar problem at the same level of difficulty.

   Sydney added 7 tenths and 1 hundredth to 34.59. What was the sum?
   
   A. 35.30
   B. 34.67
   C. 204.59
   D. 35.39

   Explain your thinking.

2. Choose a problem that is similar but slightly more challenging.

   Bart added 73 hundredths and 5 tenths to 45.68. What was the sum?
   
   A. 46.91
   B. 53.48
   C. 46.46
   D. 46.81

   Explain your thinking.
Student Work Sample: Betsy

Name ___________________________ Date __________

Sam arrived at the bus station at 11:15 p.m. but found out that his bus had left at 10:35 p.m. By how many minutes did he miss his bus?

A: 50 minutes  
B: 20 minutes  
C: 40 minutes

Explain your thinking.

Betsy: I tried to add 10:35 plus twenty because it was one of the answer choices. I got 10:55 and erased it. Then I tried to add 10:35 plus 11:15. That didn’t make too much sense because when I thought about it, the answer would be 21-something. I knew from my first attempt that twenty was too low, so I just doubled twenty to get forty.

Teacher Insights

T: Betsy used a good strategy for taking multiple-choice tests. She tried one of the answers to see if produced the desired result. In this case it did not, but it did give her a bit of direction. Continuing with this strategy might have led her to an answer that made sense to her. Instead of continuing, however, she tried to add the two times given. She did realise that the answer she produced made no sense. Then she essentially guessed. At the end of our discussion, she surmised that she could use the clock on the classroom wall as a tool to solve this question.

A Conversation with Betsy

T: Please tell me more about how you solved this problem.

Betsy: I tried to add 10:35 plus twenty because it was one of the answer choices. I got 10:55 and erased it. Then I tried to add 10:35 plus 11:15. That didn’t make too much sense because when I thought about it, the answer would be 21-something. I knew from my first attempt that twenty was too low, so I just doubled twenty to get forty.

T: How could you prove that forty works?

Betsy: I am not sure. I think I might be able to use the clock on the back wall somehow, but I’m not really sure what to count.

Informed Instructional Suggestions

Betsy’s correct answer alone would have indicated she understood the content and skills involved, but she arrived at that answer through confusion. Betsy needs opportunities to help her develop understanding and meaningful strategies for working with time. Using an analog clock would be a good way for Betsy to practice figuring elapsed time. She could also work with a drawing of a clock face or a cardboard clock face with attached minute and hour hands that she could manipulate. She did use a good test-taking strategy when she tried an answer choice, and we plan to acknowledge and reinforce this strategy.